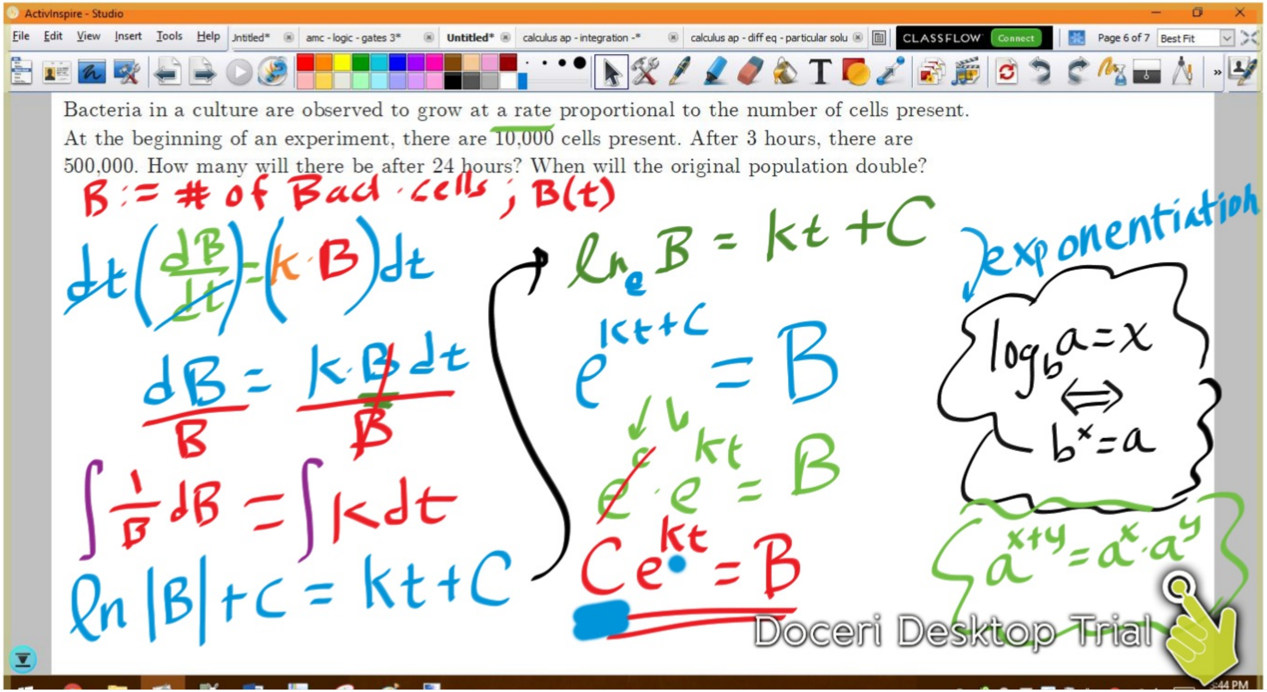
Please turn in AP Mult Choice packets into basket at end of the day tomorrow please :)

Have your notes on when the bell rings, we will continue with differential equati



$B(t) = Ce^{kt}$

initial value: (0, 10,000)

Bacteria in a culture are observed to grow at a rate proportional to the number of cells present. At the beginning of an experiment, there are 10,000 cells present. After 3 hours, there are 500,000. How many will there be after 24 hours? When will the original population double?

$10{,}000 = Ce^{k \cdot 0}$ 1

$10{,}000 = C$

$\Rightarrow B(t) = 10{,}000\, e^{kt}$

also given: $B(3) = 500{,}000$

$500{,}000 = 10{,}000\, e^{3k}$

$50 = e^{3k}$

$\ln 50 = \ln e^{3k}$

$\ln 50 = 3k$

$\frac{\ln 50}{3} = k$

$1.304 \approx k$

$B(t) = 10{,}000\, e^{1.304t}$

• Bacteria @ $t = 24$?

$B(24) = 10{,}000\, e^{(1.304)(24)}$

$= 3.906 \times 10^{17}$

• When Double?

$B(t) = 20{,}000 = 10{,}000\, e^{1.304t}$

$2 = e^{1.304t}$

$\ln 2 = 1.304t$

$\frac{\ln 2}{1.304} = t \approx 0.530$ hr

$$\frac{dy}{dx} = 2y^2$$

If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

$$\frac{dy}{2y^2} = \frac{2y^2 \cdot dx}{2y^2}$$

$$\int \frac{1}{2y^2}\, dy = \int 1\, dx$$

$$\int \frac{1}{2} \cdot y^{-2}\, dy = x + C$$

$$\frac{1}{2} \int y^{-2}\, dy = x + C$$

$$\frac{1}{2}\left[-y^{-1} + C\right] = x + C$$

$$-\frac{1}{2} y^{-1} = x + C$$

$$-2\left(-\frac{1}{2} y^{-1}\right) = (x + C)(-2)$$

$$y^{-1} = -2x + C$$

$$\frac{1}{y} = -2x + C$$

$$y = \frac{1}{-2x + C}$$

Good afternoon: we will randomize when bell rings, then complete the problem from DS

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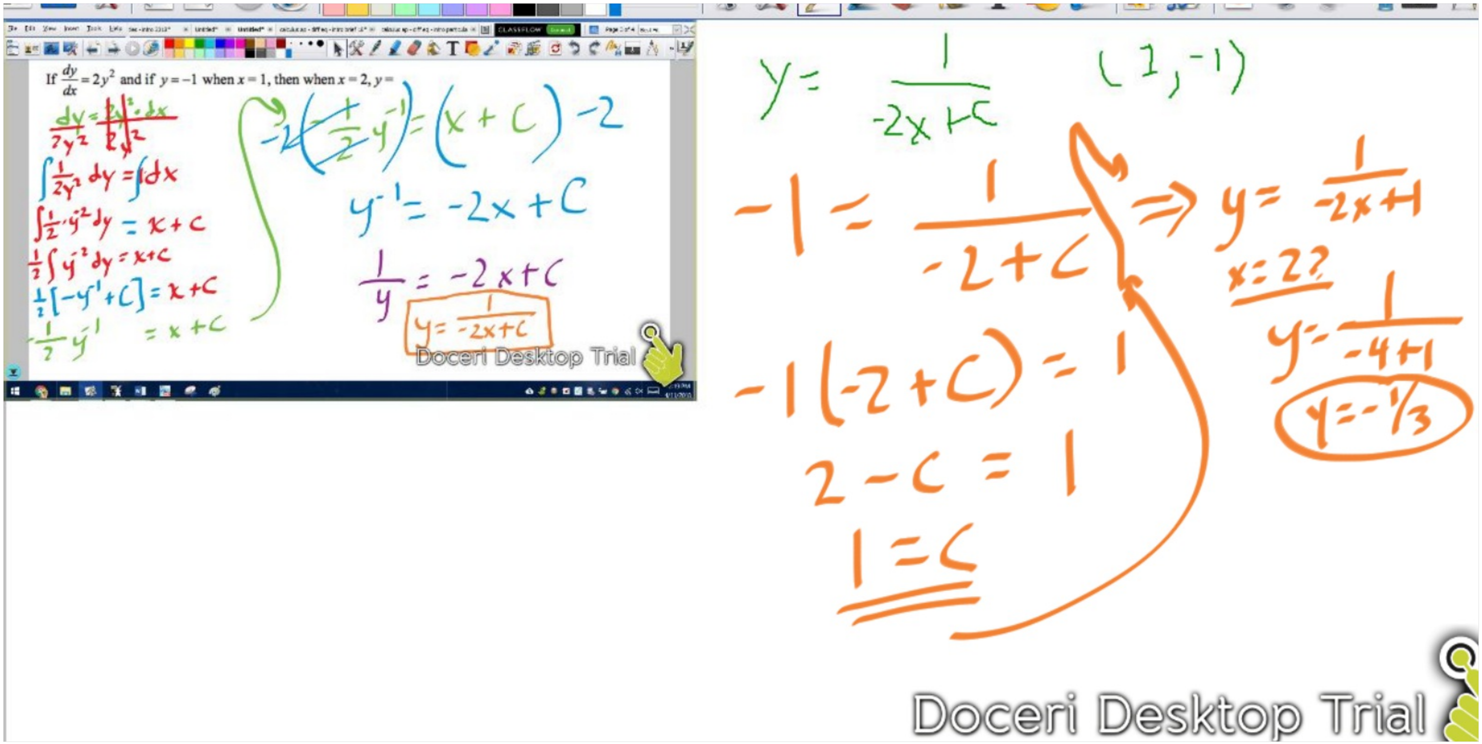
If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$
$\int \frac{1}{2y^2}\,dy = \int dx$
$\int \frac{1}{2} y^{-2}\,dy = x + C$
$\frac{1}{2}\int y^{-2}\,dy = x + C$
$\frac{1}{2}[-y^{-1} + C] = x + C$
$-\frac{1}{2} y^{-1} = x + C$
$y^{-1} = -2x + C$
$\frac{1}{y} = -2x + C$
$y = \frac{1}{-2x + C}$
Doceri Desktop Trial
$y = \frac{1}{-2x + C}$
$(1, -1)$
$-1 = \frac{1}{-2 + C}$
$-1(-2 + C) = 1$
$2 - C = 1$
$1 = C$
$\Rightarrow y = \frac{1}{-2x + 1}$
$x = 2?$
$y = \frac{1}{-4 + 1}$
$y = -\frac{1}{3}$
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A useful shortcut for some diff eq problems:

$$\frac{dY}{dt} = kY \longrightarrow Y = C\,e^{kt}$$

growth factor

initial population

e^x

$$\frac{dF}{dt} = 1.5F$$

$F(0)=12$

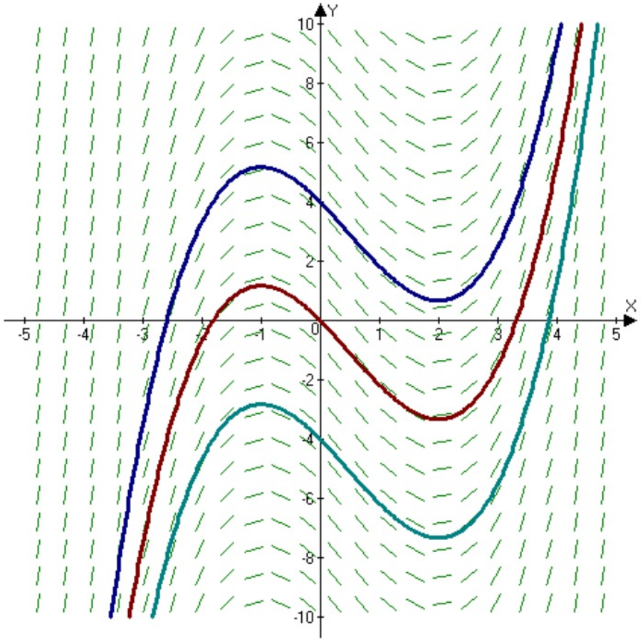
$$F(t) = 12\,e^{1.5t}$$

Slope Fields

What is the solution to a differential equation?

A family of functions (general solution)

With a given point, one can find a particular solution (finding C)

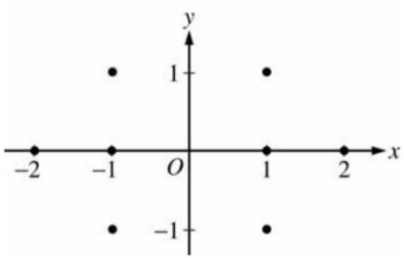


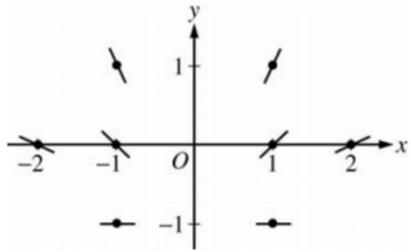
slope field represents the general solution

any particular 'channel' would be a particular solution

How to make a slope field:

1. Plug x-y coordinates of a particular point into differenial equation
2. Simplify to yield a real number
3. Sketch a little segment at that coordinate with slope equal to that number
4. Repeat with all points in the plane

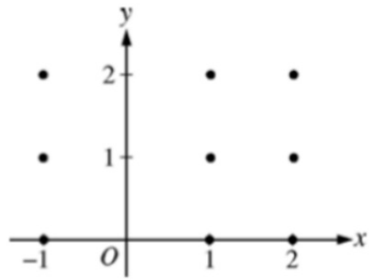




5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

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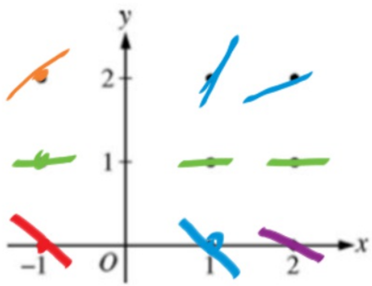
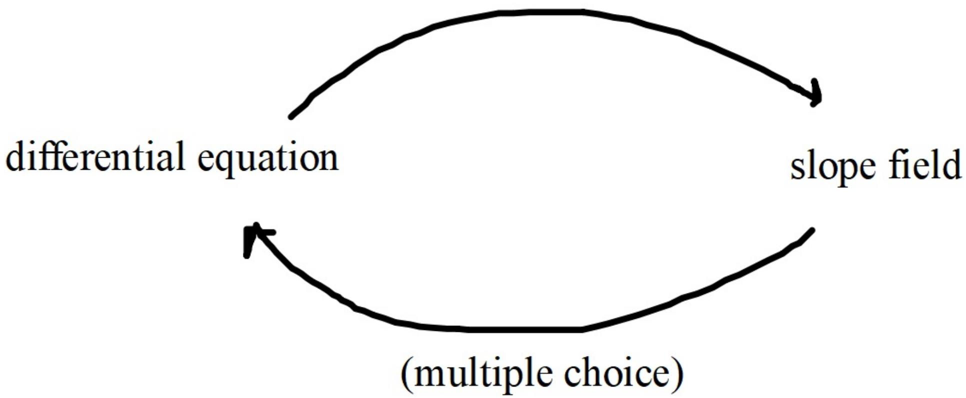


x	y	y′
−1	0	−1
−1	1	0
1	0	−1
2	0	−1/4
−1	2	1

(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

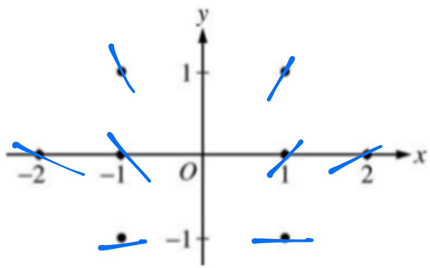
Future assessment:



5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

see below

$$\frac{dy}{dx} = \frac{1+y}{x}$$

$$\frac{x \cdot dy}{x(1+y)} = \frac{(1+y)dx}{x(1+y)}$$

$$\int \frac{1}{1+y}\,dy = \int \frac{1}{x}\,dx$$

$$\ln|1+y| = \ln|x| + C$$

$$\log_e|1+y| = \log_e|x| + C$$

$$e^{\ln x + c} = 1+y$$

$$\log_b a = x \iff b^x = a$$

$$e^{\ln x} \cdot e^{c} = 1+y$$

$$Cx = 1+y$$

$$y = Cx - 1 \quad \text{gen. solution}$$

$$f(-1) = 1$$

$$1 = C(-1) - 1$$

$$1 = -C - 1$$

$$2 = -C$$

$$-2 = C$$

$$y = -2x - 1$$

Upcoming Calendars

- Testers and Non-Testers
- Roller Coaster Project
- AP Prep Book to check out
- Note which Thursdays to stay for DS

Testers: Check your email for solutions

Important Dates

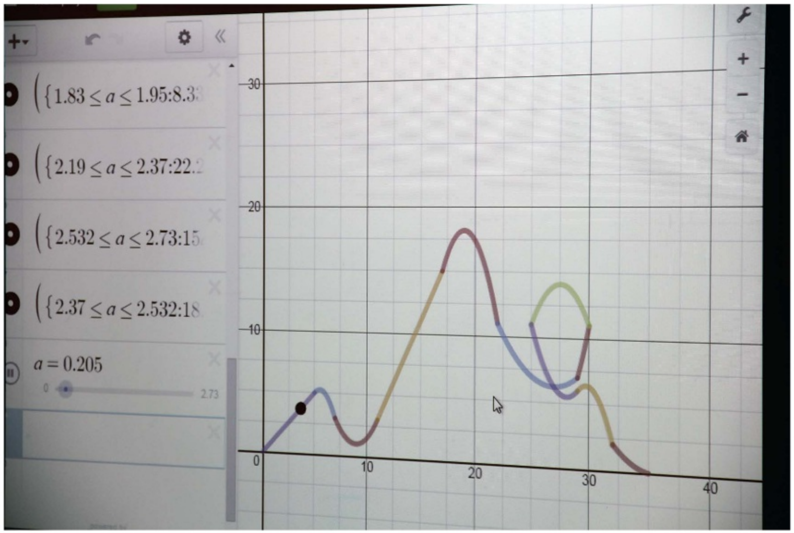
F Apr 20: Assessment on Diff Eq and Volume

F Apr 27: Take Home Assess on Related Rates, Inverses, and Optimization Due

F May 4: Roller Coaster Project Due; 2016 Mult. Choice packet due

Testers:
Saturday morning timed test...how about Apr 28th?

Non-Testers:
Note which Thursdays to stay for DS to work on/get help with project



Everyone:
Testers: last 3 timed FRQs are assssment grades
(retake involves error analysis/corrections)

Non-Testers: roller coaster project counts as 3 assessment grades

Everyone:
Calculus Party?? What date??

May 7th

Separation of Variables: Find y if

y' = 3y+6

$$\frac{dy}{dx} = 3y+6$$

$$dy = (3y+6)\,dx$$

$$dy = 3(y+2)\,dx$$

$$\frac{dy}{y+2} = 3\,dx$$

$$\int \frac{1}{y+2}\,dy = \int 3\,dx$$

$$\ln|y+2| = 3x+C$$

$$e^{3x+C} = y+2$$

$$e^{3x} \cdot e^{C} = y+2$$

$$Ce^{3x} - 2 = y$$

HW

p. 421 #3-36 (mult of 3)