$\int (3x-4)^{200} \cdot dx$

Chain Rule for Integrals

Chain: $\frac{d}{dx} F(G(x)) = F'(G(x)) \cdot G'(x)$

power-chain

$$\frac{d}{dx}\left(f(x)\right)^n = n\left(f(x)\right)^{n-1} \cdot f'(x)$$

(Power) CHAIN RULE for Integrals

$$\int f'(x) \cdot \left(f(x)\right)^n \cdot dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + C$$

this is a derivative

ex/

$$\int 3x^2(x^3+1)^5 \cdot dx$$

$$= \frac{1}{6}(x^3+1)^6 + C$$

ex/ $\int 2x \cdot \sin(x^2)\, dx = -\cos(x^2) + C$

ex/

$$\int 7x^5 \sqrt{\tfrac{7}{6}x^6+1}\; dx$$

$$= \frac{2}{3}\left(\frac{7}{6}x^6+1\right)^{3/2} + C$$

$\frac{1}{3}\int 3(3x-4)^{200} \cdot dx$

"almost $f'(x)$"

$\frac{1}{3}\int 3(3x-4)^{200}\, dx$

$\frac{1}{3}\left[\frac{1}{201}(3x-4)^{201} + C\right]$

$\frac{1}{603}(3x-4)^{201} + C$

Scalar multiple

$\int a \cdot f(x)\,dx = a\int f(x)\,dx$

ex $\int \frac{1}{3} 3 \cdot (3x-4)^{200}\, dx$

$\frac{1}{3}\int 3(3x-4)^{200}\, dx$

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ex/

$$\frac{1}{3}\int 3x^2 \cdot \cos(x^3) \cdot dx = \frac{1}{3}\sin(x^3) + C$$

"almost $f'(x)$"

"Finding C"

F(x) is the derivative of G(x).

$G(0)=3$.

$F(x) = 3x^2+5$. What is $G(x)$?

$$G'(x) = F(x)$$
$$G'(x) = 3x^2+5$$
$$\cancel{dx}\left(\frac{dg}{\cancel{dx}} = 3x^2+5\right)dx$$ (Separation of variables)
$$\int 1\,dg = \int 3x^2+5\,dx$$
$$g = x^3+5x+C$$
$$g(0)=3$$
$$g(0)=3=0^3+5(0)+C$$
$$3=C$$
$$\boxed{g = x^3+5x+3}$$

ex/ $f'(x) = 3\sin(6x)$, $\boxed{f(0)=6}$

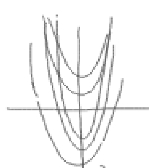
What is $f(x)$?

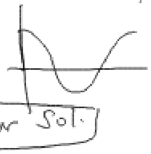
$$\cancel{dx}\left(\frac{df}{\cancel{dx}}\right) = \left(3\sin(6x)\right)dx$$
$$\int df = \int 3\sin(6x)\cdot dx$$
$$f + C = \frac{1}{2}\int 2\cdot 3\sin(6x)\cdot dx$$

"almost fit"

$$f + d = \frac{1}{2}\int 6\sin(6x)\cdot dx$$
$$f + d = \frac{1}{2}\left(-\cos(6x)+C\right)$$
$$f + \cancel{d} = -\frac{1}{2}\cos(6x)+C$$
$$\underline{-d} \qquad \underline{-c}$$
$$f = -\frac{1}{2}\cos(6x)+C$$

general solution





$$f(0)=6=-\frac{1}{2}\cos(0)+C$$
$$6=-\frac{1}{2}\cdot 1+c$$
$$6=-\frac{1}{2}+c$$
$$6.5=c$$

Particular Sol.

$$\boxed{f(x) = -\frac{1}{2}\cos(6x)+6.5}$$