Inverse Trig Derivatives

Set $y = \sin^{-1}(x)$. Find $\frac{dy}{dx}$

$\frac{d}{dx}(x) = (\sin(y))\frac{d}{dx}$ inverse trigs always equal <u>angles</u>.

$$1 = \cos(y)\,\frac{dy}{dx}$$

$$\frac{1}{\cos(y)} = \frac{dy}{dx}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\sqrt{\cos^2 y} = \sqrt{1-\sin^2 y}$$

$$\cos y = \sqrt{1-\sin^2 y}$$

$$\frac{1}{\sqrt{1-\sin^2 y}} = \frac{dy}{dx}$$

$x = \sin y$

$$\frac{1}{\sqrt{1-x^2}} = \frac{dy}{dx}$$

$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

Inverse Trig derivative example

ex

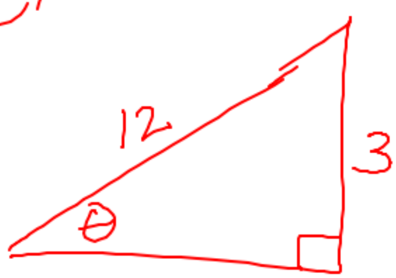
$$\frac{d}{dx}\sin^{-1}\left(3e^{x^2}\right)$$

$$= \frac{1}{\sqrt{1-(3e^{x^2})^2}} \cdot 3e^{x^2} \cdot 2x$$

$$\frac{6xe^{x^2}}{\sqrt{1-9e^{x^4}}}$$

Review: what is even an inverse trig function?

ex



Find θ.

$$\cancel{\sin^{-1}}\left(\cancel{\sin}\,\theta = \frac{3}{12}\right)\sin^{-1}$$

$$\theta = \sin^{-1}\left(3/12\right)$$

$$\theta \approx 14.47^\circ$$

10.) $\int \sin 5x \cdot \underbrace{\cos(5x)} \, dx$

Let $u = \cos(5x)$

$\frac{du}{dx} = -5\sin(5x)$

$du = -5 \sin 5x \cdot dx$

$\frac{du}{-5 \sin 5x} = dx$

$\int \sin 5x \cdot u \cdot \frac{du}{-5 \sin 5x}$

$\int \frac{u \cdot \cancel{\sin 5x} \cdot du}{-5 \cancel{\sin 5x}}$

$\int -\frac{1}{5} \cdot u \cdot du$

$-\frac{1}{5} \int u \cdot du$

$-\frac{1}{5}\left(\frac{1}{2}u^2 + C \right)$

$-\frac{1}{10}u^2 + C$

$\boxed{-\frac{1}{10}\cos^2(5x) + C}$