

Inverse Trig Derivatives

Set $y = \sin^{-1}(x)$. Find $\frac{dy}{dx}$

$\frac{d}{dx}(x) = (\sin(y)) \frac{dy}{dx}$ inverse trigs always equal angles.

$$1 = \cos(y) \frac{dy}{dx}$$

$$\frac{1}{\cos(y)} = \frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-\sin^2 y}} = \frac{dy}{dx}$$

$$x = \sin y$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{dy}{dx}$$

$$\begin{aligned} \sin^2 y + \cos^2 y &= 1 \\ \sqrt{\cos^2 y} &= \sqrt{1-\sin^2 y} \\ \cos y &= \sqrt{1-\sin^2 y} \end{aligned}$$

$$\frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

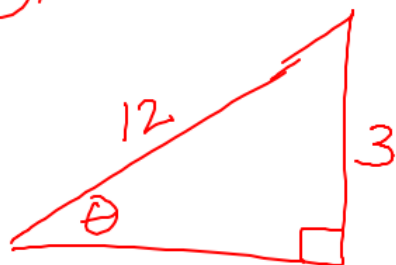
$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

Inverse Trig derivative example

$$\begin{aligned} & \frac{d}{dx} \sin^{-1}(3e^{x^2}) \\ &= \frac{1}{\sqrt{1-(3e^{x^2})^2}} \cdot 3e^{x^2} \cdot 2x \\ &= \frac{6xe^{x^2}}{\sqrt{1-9e^{2x^2}}} \end{aligned}$$

Review: what is even an inverse trig function?

ex



$$\begin{aligned} & \sin^{-1} \left(\sin \theta = \frac{3}{12} \right) \sin^{-1} \\ & \theta = \sin^{-1} \left(\frac{3}{12} \right) \end{aligned}$$

Find θ .

$$\theta \approx 14.47^\circ$$

Integrals of Varying Difficulty: examples

$$10.) \int 5 \sin 5x \cdot \underline{\cos(5x)} dx$$

$$\text{Let } u = \cos(5x)$$

$$\frac{du}{dx} = -5 \sin(5x)$$

$$du = -5 \sin 5x dx$$

$$\frac{du}{-5 \sin 5x} = dx$$

$$\int \sin 5x \cdot u \cdot \frac{du}{-5 \sin 5x}$$

$$\int \frac{u \cdot \cancel{\sin 5x} \cdot du}{-5 \cancel{\sin 5x}}$$

$$\int -\frac{1}{5} \cdot u \cdot du$$

$$-\frac{1}{5} \int u \cdot du$$

$$-\frac{1}{5} \left(\frac{1}{2} u^2 + C \right)$$

$$-\frac{1}{10} u^2 + C$$

$$\boxed{-\frac{1}{10} \cos^2(5x) + C}$$