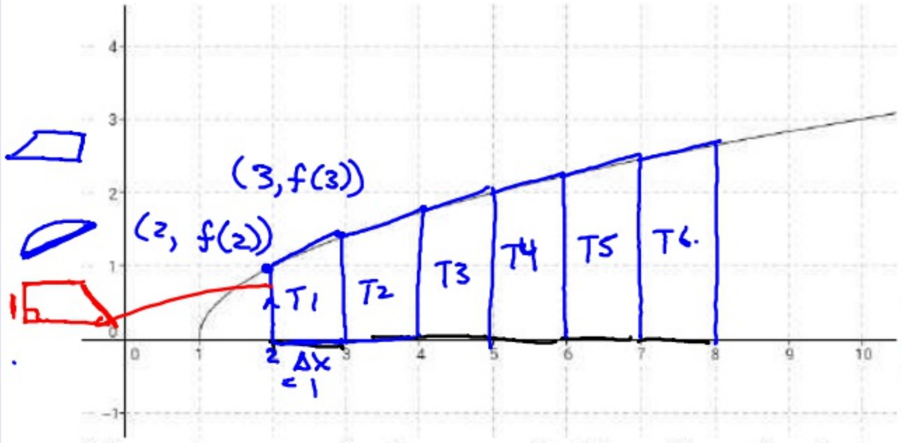
Trapezoidal Approximation Method

Recall: LRAM, RRAM, and MRAM are three Riemann sum approximation methods involving rectangles. LRAM uses the left endpoint of a given interval, RRAM uses the right endpoint, and MRAM uses the midpoint to define the height. Whichever height you use, the Riemann sum formula for area under a curve from [a,b] using n rectangles:

$$A \approx \sum_{i=1}^{n} f(x_i)\Delta x \qquad \textit{where } \Delta x = \frac{b-a}{n}$$

$\Delta x = \frac{8-2}{6} = \frac{6}{6} = 1$

Another approximation method uses trapezoids.
Goal: Approximate the area under $f(x) = \sqrt{x-1}$ over the interval [2, 8] using 6 trapezoids.



1. Draw the trapezoids.

2. Area = (average of bases)*(height)
   What are the bases?
   What is the height?

$A_{T1} = (f(2)+f(3))\frac{1}{2}\cdot 1$

$A_{T2} = (f(3)+f(4))\frac{1}{2}\cdot 1$

$A_{T3} = (f(4)+f(5))\frac{1}{2}\cdot 1$

3. Express the area approximation as a sum of each trapezoid area: (you do not need to use summation notation)

$$A \approx (f(2)+f(3))\cdot\frac{1}{2}\cdot 1 + (f(3)+f(4))\frac{1}{2}\cdot 1 + (f(4)+f(5))\frac{1}{2}\cdot 1 + (f(5)+f(6))\frac{1}{2}\cdot 1 + (f(6)+f(7))\cdot\frac{1}{2}\cdot 1 + (f(7)+f(8))\cdot\frac{1}{2}\cdot 1$$

4. Can you factor anything out of this sum? Do so:

$$\frac{1}{2}\cdot\underbrace{1}_{\frac{b-a}{n}=\Delta x}\left(f(2)+f(3)+f(3)+f(4)+f(4)+f(5)+f(5)+f(6)+f(6)+f(7)+f(7)+f(8)\right)$$

$f(x) = \sqrt{x-1}$

5. Can you simplify what is left inside the parentheses? Then find the area, and compare it to the exact value.

$$\frac{1}{2}\cdot 1\left(f(2)+2\left(f(3)+f(4)+f(5)+f(6)+f(7)\right)+f(8)\right)$$

$$\frac{1}{2}\left(1+2\left(\sqrt{2}+\sqrt{3}+2+\sqrt{5}+\sqrt{6}\right)+\sqrt{7}\right) \approx 11.65$$

Trapezoidal Rule: The area under the curve f(x) from [a,b] using n trapezoids is:

$$A = \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right)$$

In simple English: multiply all the y-values *except* the first and last by 2, add them all together, multiply by $\frac{\Delta x}{2}$

2011AB2: calculator active

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?