$$2\int \tfrac{1}{2}\, 2\sqrt[3]{x-1}\,dx \quad \underline{vs.} \quad \int 2x\sqrt[3]{x-1}\,dx$$

$$\int 7x^5\sqrt{\tfrac{7}{6}x^6+1}\,dx$$

$$2\int 1\sqrt[3]{x-1}\,dx = 2\int 1\cdot(x-1)^{1/3}\,dx$$

$$2\cdot\frac{3}{4}(x-1)^{4/3}$$

$$\frac{3}{2}(x-1)^{4/3}+C$$

$\int 2x \sqrt[3]{x-1}\, dx$

$\boxed{u\text{-substitution}}$

↳ when Reverse chain rule fails

Set $u = g(x)$ if $\int f(g(x))\, dx$

Let $u = x-1$. → so $x = u+1$

→ so,

$$\int \underbrace{2(u+1)}_{x} \underbrace{\sqrt[3]{u}}_{x-1} \underbrace{du}_{dx}$$

$\frac{d}{dx}(u) = (x-1)\frac{d}{dx}$

$\frac{du}{dx} = 1 \to du = dx$

$$\int (2u+2)u^{1/3}\, du$$

$$\int 2u^{4/3} + 2u^{1/3}\, du$$

$$2 \cdot \frac{3}{7}u^{7/3} + 2 \cdot \frac{3}{4}u^{4/3} + C$$

$$\frac{6}{7}u^{7/3} + \frac{3}{2}u^{4/3} + C \; ; \; u = x-1$$

$$\frac{6}{7}(x-1)^{7/3} + \frac{3}{2}(x-1)^{4/3} + C \quad **$$

$$(x-1)^{4/3}\left(\frac{6}{7}(x-1)^{1} + \frac{3}{2}\right) + C$$

$$\left(\frac{6}{7}x - \frac{6}{7} + \frac{3}{2}\right) \quad -\frac{12}{14} + \frac{21}{14}$$

$$(x-1)^{4/3}\left(\frac{12}{14}x - \frac{9}{14}\right)$$

$$\frac{3}{14}(x-1)^{4/3}(4x - 3) + C$$