4.4 Using Geometry for Definite Integrals

Graph the integrands and use geometry to evaluate the definite integrals.

911. $\int_{-2}^{4} \left(\frac{x}{2}+3\right)\, dx = 21$ trapez.

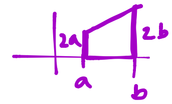
912. $\int_{-3}^{3} \sqrt{9-x^2}\, dx = \frac{9\pi}{2}$ Semi-circle

913. $\int_{-2}^{1} |x|\, dx = \frac{5}{2}$ triangles

914. $\int_{-1}^{1} (2-|x|)\, dx = 3$ 2 trapez.

915. $\int_{0}^{b} x\, dx$ where $b>0$ $= \frac{1}{2}b^2$ triangle

916. $\int_{a}^{b} 2x\, dx$ where $0<a<b$



$= \overbrace{\frac{1}{2}(2a+2b)}^{\text{Avg. Bases}}\overbrace{(b-a)}^{\text{height}}$

$(a+b)(b-a)$

$\cancel{ab} - a^2 + b^2 - \cancel{ab} = b^2 - a^2$

917. Suppose f and g are continuous and that

$$\int_1^2 f(x)\, dx = -4, \qquad \int_1^5 f(x)\, dx = 6, \qquad \int_1^5 g(x)\, dx = 8.$$

Evaluate the following definite integrals.

a) $\int_2^2 g(x)\, dx = 0$

b) $\int_5^1 g(x)\, dx = -8$

c) $\int_1^2 3f(x)\, dx$ -12

d) $\int_2^5 f(x)\, dx = 10$

$\int_1^5 f(x)dx - \int_1^2 f(x)$

e) $\int_1^5 [f(x)-g(x)]\, dx = -2$

f) $\int_1^5 [4f(x)-g(x)]\, dx$

$24 - 8 = 16$

918. Suppose that $\int_{-3}^{0} g(t)\, dt = \sqrt{2}$. Find the following.

a) $\int_0^{-3} g(t)\, dt = -\sqrt{2}$

b) $\int_{-3}^{0} g(u)\, du = \sqrt{2}$

c) $\int_{-3}^{0} -g(x)\, dx = -\sqrt{2}$

d) $\int_{-3}^{0} \frac{g(\theta)}{\sqrt{2}}\, d\theta = \frac{1}{\sqrt{2}}\int_{-3}^{0} g(\theta)\, d\theta$

$\frac{1}{\sqrt{2}}\cdot\sqrt{2} = 1$

919. A particle moves along the x-axis so that at any time $t \ge 0$ its acceleration is given by $a(t) = 18-2t$. At time $t=1$ the velocity of the particle is 36 meters per second and its position is $x=21$.

$V(1) = 36$ $\qquad X(1) = 21$

a) Find the velocity function and the position function for $t \ge 0$.

b) What is the position of the particle when it is farthest to the right?

a.) $v(t) = \int a(t)dt = \int 18-2t\ dt = 18t - t^2 + C$

$v(t) = 18t - t^2 + C$

$v(1) = 18 - 1 + C = 36 \Rightarrow C = 19$

$\boxed{v(t) = 18t - t^2 + 19}$

$x(t) = \int v(t)dt$

$= \int 18t - t^2 + 19\ dt$

$= 9t^2 - \frac{1}{3}t^3 + 19t + C$

$x(1) = 21$

$9 - \frac{1}{3} + 19 + C = 21$

$\Rightarrow C = \frac{-20}{3}$

$x(t) = -\frac{1}{3}t^3 + 9t^2 + 19t - \frac{20}{3}$

When you feel how depressingly
Slowly you climb,
It's well to remember
That things take time.

—*Piet Hein*

$x(19)$

$= 1317$ m

b.) Farthest right = max position

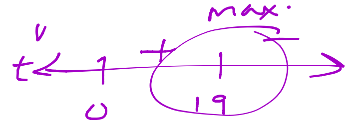
max position $\Rightarrow$ $v(t) = 0$ (rel. max)

$0 = -t^2 + 18t + 19$

$0 = -(t^2 - 18t - 19)$

$0 = -(t-19)(t+1)$

$t = 19$ $\quad \cancel{t = -1}$ Domain $(t \ge 0)$



$t = 19$