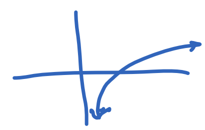
AP Calculus Study Guide: Indefinite Integrals Test

About 30 questions; some multiple choice, some short answer; no calculator and yes calculator sections

- Evaluate the indefinite integral for the following function types:
  - Polynomial with integer exponents: $\int 3x^5 + 2x^3 + x^{-2}dx$; $\int (x^3+1)^2 dx$
  - Function with radicals that can be made into a polynomial $\int \sqrt[7]{x^3}(x-5)dx$
  - Rational function that can be made into a polynomial $\int \frac{7x^5-2x^3+x}{\sqrt{x}}dx$



  - Rational function that is actually an ln problem: $\int \frac{1}{3x}dx$ =

$\int \frac{1}{3x}dx \to \int \frac{1}{3}\cdot\frac{1}{x}dx \to \frac{1}{3}\int \frac{1}{x}dx \to \frac{1}{3}\ln|x| + C$

  - $\boxed{\text{Trigonometric}}$ functions $\int \sec^2(4x)\,dx$

$\frac{1}{4}\int 4\sec^2(4x)\,dx \to \boxed{\frac{1}{4}\tan(4x) + C}$

  - Reverse chain rule involving polynomials $\int x(3x^2+1)^{75}dx$; $\int 12x^2\sqrt[3]{4x^3-2}\,dx$ $\to \frac{3}{4}(4x^3-2)^{4/3}+C$

$\frac{1}{6}\int 6x(3x^2+1)^{75} \to \frac{1}{6}\cdot\frac{1}{76}(3x^2+1)^{76}+C \to \frac{1}{456}(3x^2+1)^{76}+C$

  - Reverse chain rule involving e and ln(x) $\int \frac{-6x^2}{e^{2x^3}}dx$; $\int \frac{6x^2}{x^3+12}dx$

$\int -6x^2\cdot e^{-2x^3}dx = e^{-2x^3}+C$ $\quad\to 2\ln|x^3+12|+C$

  - Reverse chain rule involving trigonometric functions: $\int 4x^5 * \sin(12x^6)\,dx$

$\frac{1}{28}(2x-1)^7 + \frac{7}{24}(2x-1)^6 + C$

$-\frac{1}{18}\cos(12x^6)+C$ $\quad \frac{1}{18}\int 18\cdot 4x^5\sin(12x^6)\,dx$

  - U-substitution for when reverse chain rule fails: $\int (x+3)(2x-1)^5dx$ Let $u = 2x-1$.

- Finding C: given a derivative and an initial condition, find the particular solution with a value for C.

$\int \frac{2}{t} + 4t^2 - 3e^{3t}\,dt$

Ex: A particle moves along the x-axis such that its velocity is given by $v(t) = \frac{2}{t} + 4t^2 - 3e^{3t}$. At time t=1, the particle's position is x(1) = 5. Find *x(t)*.

$2\ln|t| + \frac{4}{3}t^3 - e^{3t} + C = x(t)$

$x(1) = 5 = \cancel{2\ln(1)}^{0} + \frac{4}{3}(1) - e^3 + C$

$x(t) = 2\ln|t| + \frac{4}{3}t^3 - e^{3t} + \left(\frac{11}{3} + e^3\right)$

$5 = 0 + \frac{4}{3} - e^3 + C \Rightarrow \boxed{C = \frac{11}{3} + e^3}$

Review topics:

- Related rates without a variable substitution: (will be provided geometry formulas if they're not obvious)

$A = \pi r^2$ $\quad \frac{dr}{dt} = 1$

Ex: A pebble falls in a pond and causes a circular ripple. The radius of the ripple is increasing by 1cm per second. At the instant the circumference is equal to 12pi, how fast is the area changing?

$C = \boxed{12\pi = 2\pi r}$ $\quad r = 6$ $\quad \frac{dA}{dt} - ?$

$\frac{d}{dt} A = (\pi r^2)\frac{d}{dt}$

$\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt}$

$\frac{dA}{dt} = 12\pi \cdot 1$

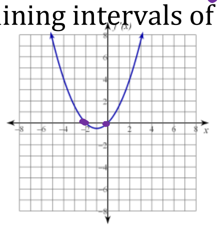
12π cm²/sec

- Finding limits at a discontinuity without a calculator
  - $\lim_{x\to 5^-} \frac{2x^2-50}{x-5}$ $\rightarrow \frac{2(x^2-25)}{x-5} \rightarrow \frac{2(x+5)\cancel{(x-5)}}{\cancel{x-5}}$ $\lim_{x\to 5^-} 2(x+5) = 20^-$ $\quad 20$

- Using implicit differentiation to find dy/dx:
  - Given that $x^2 + xy + y^2 = 1$ Show that dy/dx is $\frac{-2x-y}{x+2y}$

$2x + \left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$

- Determining intervals of increasing/decreasing and concavity given a graph of F'



f'' neg $\rightarrow f'$ is decreas

Given the graph of F', when is F increasing? When is it concave down?

$f' > 0$ $\quad (-\infty, -1)$

$(-\infty, -2) \cup (0, \infty)$

- Finding and justifying extrema
  - Given f(x) = $-x^4 + 2x^2 - 1$, find the points that are local maximums. Justify your answer.

Set $f'(x) = 0$ $\quad x = 0, -x^2 + 1 = 0$ $\quad$ test values

$4x^3 + 4x = 0$ $\quad x = 0$ $\quad x = \pm 1$

$4x(-x^2 + 1) = 0$ $\quad x$

MAX MIN MAX

$+ \quad - \quad + \quad -$

$-1 \quad 0 \quad 1$

- Mean Value Theorem: find the value of c that satisfies the MVT for a given differentiable function
  - Let *f* be the function given by $f(x) = x^3 - 7x + 6$. Find the number *c* that satisfies the conclusion of the Mean Value Theorem for *f* on the closed interval [1, 3].

$f(3) = 12$

$f(1) = 0$

Average Slope = Instant Slope (Derivative)

$\frac{f(3) - f(1)}{3 - 1} = 3x^2 - 7$

$\frac{12 - 0}{2}$

$6 = 3x^2 - 7$

Solve

$6 = 3x^2 - 7$

$13 = 3x^2$

$\frac{13}{3} = x^2$

$\pm\sqrt{\frac{13}{3}} = x$

only the pos answer is in [1,3]

So, $c = \sqrt{13/3}$

Another u-sub example:

$$\int x\sqrt{2x+1}\,dx \longrightarrow \int \frac{u-1}{2}\cdot(u)^{1/2}\cdot\frac{du}{2}$$

$\times 2$

Let $\boxed{u=2x+1}$

therefore $\boxed{x=\frac{u-1}{2}}$

$$\frac{du}{dx}=2$$

$$du=2\,dx$$

$$\boxed{\frac{du}{2}=dx}$$

$$\int \frac{1}{2}(u-1)\,u^{1/2}\cdot\frac{1}{2}\cdot du$$

$$\int \frac{1}{4}(u-1)\cdot u^{1/2}\cdot du$$

$$\frac{1}{4}\int (u-1)\,u^{1/2}\,du$$

$$\frac{1}{4}\int u^{3/2}-u^{1/2}\cdot du$$

$$\frac{1}{4}\left[\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}+C\right]$$

$$\frac{2}{20}u^{5/2}-\frac{1}{6}u^{3/2}+C$$

$$\frac{1}{10}(2x-1)^{5/2}-\frac{1}{6}(2x-1)^{3/2}+C$$

Trig anti derivative involving reverse chain rule

ex

$$-\frac{1}{2}\int -2\csc(2x)\cot(2x)\,dx$$

$$-\frac{1}{2}\csc(2x) + C$$

Rationals involving an ln

$$\int \frac{4x}{2x^2+1}\,dx$$

$$\int 4x \cdot \frac{1}{2x^2+1}\,dx$$

$$\ln|2x^2+1| + C$$

$$\int \tan(x)\cdot dx$$

$$\int \frac{\sin(x)}{\cos(x)}\cdot dx$$

$$-\int -\sin(x)\cdot\frac{1}{\cos(x)}\cdot dx$$

$$-\ln|\cos(x)| + C$$

$+\frac{1}{\cos}$ $+\sin \rightarrow \tan$