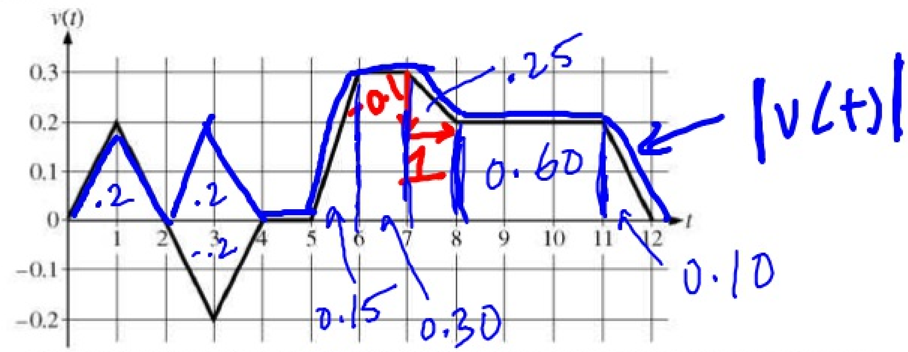
2009AB1



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

a.) $a(t) = v'(t)$ (slope of v)

$a(7.5) = v'(7.5) = \frac{\Delta y}{\Delta x} = \frac{-0.1 \text{ mi/min}}{1 \text{ min}}$

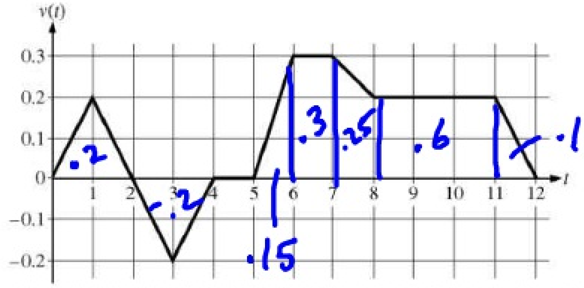
$= \boxed{-0.1 \text{ mi/min}^2}$

b.) Total distance Caren travels in time 0 to 12 min. Absolute value has effect of counting negative velocity (backwards movement) as positive distance traveled.

$0.2 + 0.2 + 0.15 + 0.3 + 0.6 + 0.1 = 1.8$ miles

c.) $t = 2$ min. vel changes sign, so Caren changes direction

2009AB1



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d.) Caren is at home at $t = 5$. She leaves and goes directly to School. Distance traveled?

$$\int_5^{12} v(t)\,dt = .15 + .3 + .25 + .6 + .1$$
$$= 1.4 \text{ miles}$$

=or=

$$\int_0^{12} v(t)\,dt = 0.2 + -0.2 + 0.15 + 0.3 + 0.25 + 0.6 + 0.1$$

(note no abs. value)

$$= 1.4\text{ mi}$$

LARRY

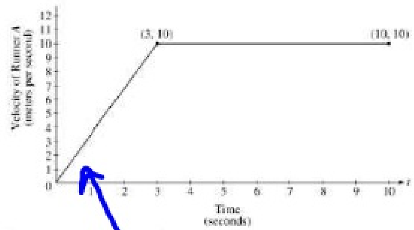
$$\int_0^{12} w(t)\,dt \Rightarrow \underbrace{\int_0^{12} \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)dt}_{\text{Math-9}}$$

$$\approx 1.6$$

$$\boxed{\text{Caren, } 1.4 < 1.6}$$

2000AB2 calc ok

Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \frac{24t}{2t+3}$.



(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

(d) 5 seconds into the race, who is winning, runner A or runner B? Justify your response.

a.) $V_a(2) \Rightarrow$ need equation of line

Slope: $\frac{10-0}{3-0} = \frac{10}{3}$

$$V_a(t) = \frac{10}{3}t + 0$$

$$V_a(2) = \frac{10}{3} \cdot 2 = \frac{20}{3} = \boxed{6.667 \text{ m/s}}$$ A

$$V_B(2) = \frac{24(2)}{2(2)+3} = \frac{48}{7} \approx 6.857 \text{ m/s}$$ B

b.) $acc_a(t) = V_a'(t)$

$V_a(t) = \frac{10}{3}t$, so $V_a'(t) = \frac{10}{3}$

so $a_a(t) = \frac{10}{3}$

and $\boxed{a_a(2) = \frac{10}{3} \text{ m/s}^2}$

$$V_B'(2) = a_b(2)$$

$= \boxed{\text{MATH } +8} \rightarrow \frac{d}{dx}\left[\frac{24x}{2x+3}\right]_{x=2}$

beep boop ... $= \boxed{1.469 \text{ m/s}^2}$

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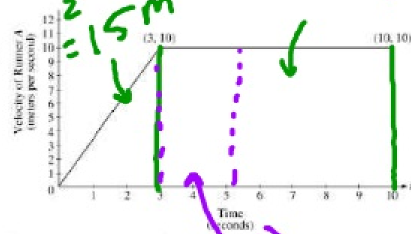

$\frac{1}{2} \cdot 3 \cdot 10 = 15m$

$7 \cdot 10 = 70m$

d.) $2 \cdot 10 = 20$

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(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

(d) 5 seconds into the race, who is winning, runner A or runner B? Justify your response.

c.) A: $\int_0^{10} V_a(t)\,dt$ = area of trapezoid (or $\triangle + \square$)

$= \boxed{85\ m}$ A

B. $\int_0^{10} \frac{24x}{2x+3}\,dx \to$ MATH 9 $\to \boxed{83.336m}$ B

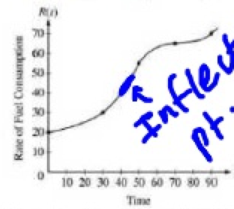
d.) $\int_0^5 V_a(t)\,dt = 15 + 20 = \underline{35m.}$

$\int_0^5 \frac{24x}{2x+3}\,dx \to$ math 9 $\to \underline{33.606m}$ B

Runner A is farther along.

2003AB3 NC **No Calc**

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.



Inflection pt.

t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) Explain the meaning of $\int_0^{90} R(t)dt$ in the context of the problem. Include units in your response.

a.) $R'(45)$ slope of R @ $t=45$

$$\frac{\Delta y}{\Delta x} = \frac{55-40}{50-40} \frac{\text{g/min}}{\text{min}}$$

$$\frac{15}{10}\ \text{g/min}^2$$

$$1.5\ \text{g/min}^2$$

b.)

"$\underbrace{\text{Rate of fuel consumption}}$ $\underbrace{\text{increasing}}$ $\underbrace{\text{fastest}}$"

$R(t) \rightarrow R'(t) \rightarrow R'(t)$ max

where does $R'(t)$ have extrema?

where $R''(x) = 0$ [critical numbers of R', or Inf. Pt of R]

thus, $\underline{R''(45) = 0}$.

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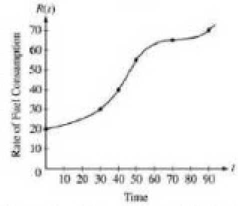

t (minutes)	$R(t)$ (gallons per minute)
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c.) Note that Δt is not constant!

t	0	30	40	50	70	90
$R(t)$	20	30	40	55	65	70

Δt 30 10 10 20 20

LRAM

$$\int_0^{90} \approx \underbrace{30\cdot 20}_{\text{min} \quad \text{g/min}} + \underbrace{10\cdot 30} + \underbrace{10\cdot 40} + \underbrace{20\cdot 55} + \underbrace{20\cdot 65}$$

$$\approx 600 + 300 + 400 + 1100 + 1300$$

gal.

$$\begin{array}{r} 55 \\ \times 20 \\ \hline 1100 \end{array}$$

$$\approx 900 + 400 + 1100 + 1300$$

2000 1700

$$\boxed{3700 \text{ gal}}$$

d.) Approximation for total amount, in gallons, of fuel consumed over first 90 min of flight.