Good afternoon: assessments are being returned.
Overall, pretty good!

All skills were new--most will be assessed again
on the next test (to make a 96 → 100 hopefully!)

# A loose end: properties of definite integrals

$\int_{12}^{-10} f(x)\, dx = 6$, $\boxed{\int_{100}^{-10} f(x)\, dx = -2}$ and $\int_{100}^{-5} f(x)\, dx = 4$ determine the value of $\int_{-5}^{12} f(x)\, dx$.

- $\int_a^b = -\int_b^a$
- $\int_a^b = \int_a^c + \int_c^b$

$$\int_{-5}^{100} + \int_{100}^{-10} + \int_{-10}^{12}$$

$$\int_{-5}^{12} = -\int_{12}^{-5}$$

$$\int_{-10}^{-5}$$

$$-\left[\int_{12}^{-10} + \underbrace{\int_{-10}^{100} + \int_{100}^{-5}}\right]$$

$$-\left[6 \cancel{-\int_{100}^{-10}} + 4\right]$$

$$+2$$

$$-12$$

# Area

Find the area of the region

$$f(x) = \begin{cases} x^3 - 2x + 4 & x \le 0 \\ 4 - x^2 & x > 0 \end{cases}$$

A $\displaystyle\int_{-2}^{0} (x^3 - 2x + 4)\,dx$

$$= \left[\frac{1}{4}x^4 - x^2 + 4x\right]_{-2}^{0}$$

$$(0) - (-8)$$

$$8$$

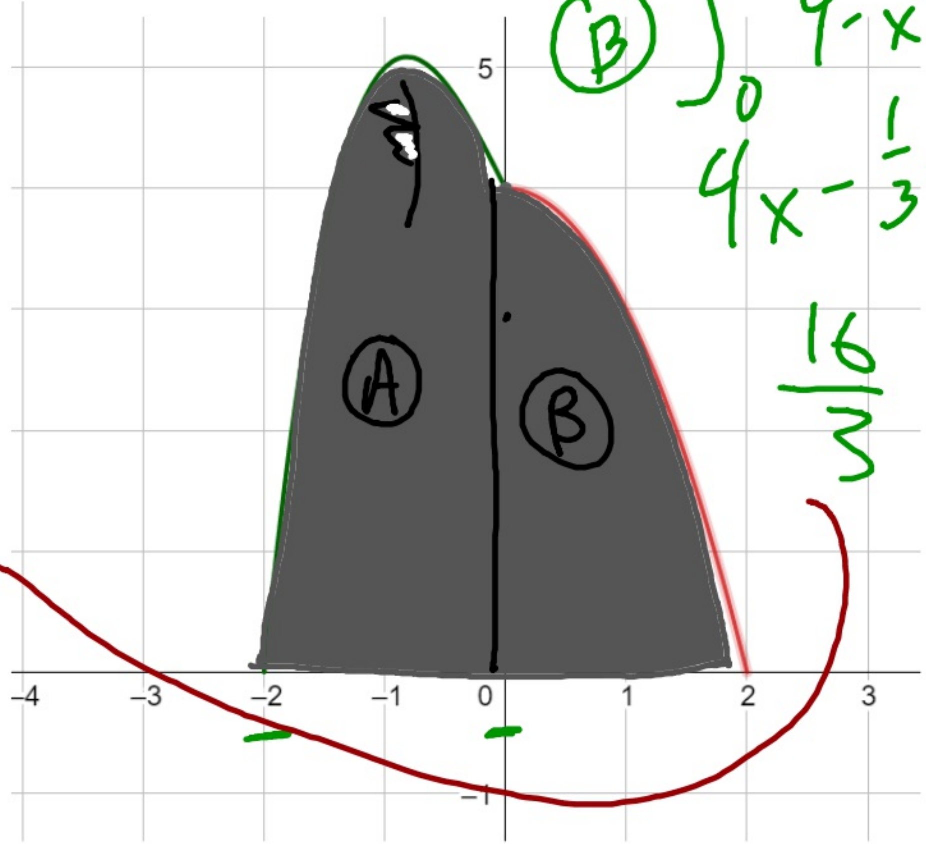
$$8 + \frac{16}{3}$$

$$\frac{40}{3}$$

B $\displaystyle\int_{0}^{2} 4 - x^2\,dx$

$$4x - \frac{1}{3}x^3 \Big|$$

$$\frac{16}{3}$$



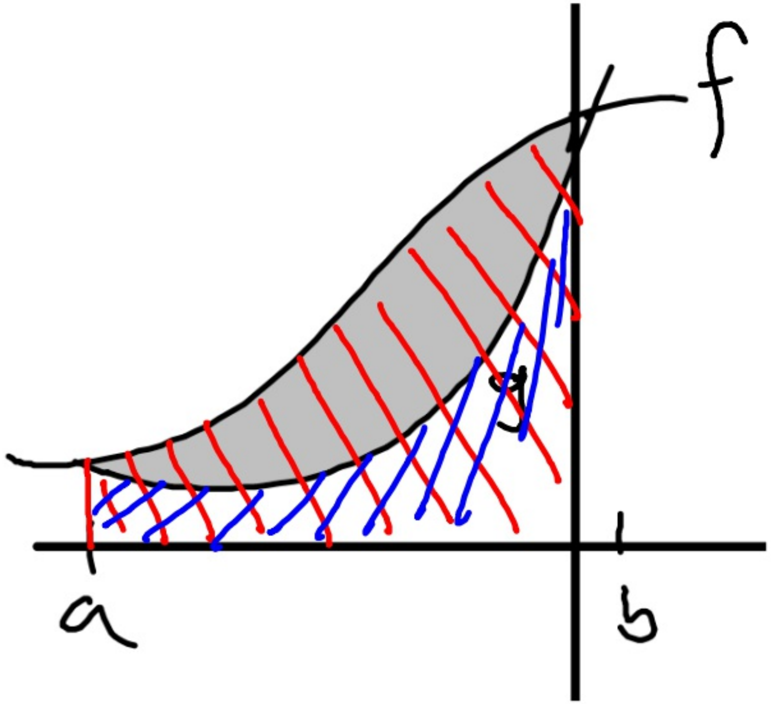
Area under a curve is pretty straightforward...
It's what the Riemann definite integral definition is all about!

More interesting:

to find area of grey
find the area under top curve (red)
then subtract area under bottom curve (blue)

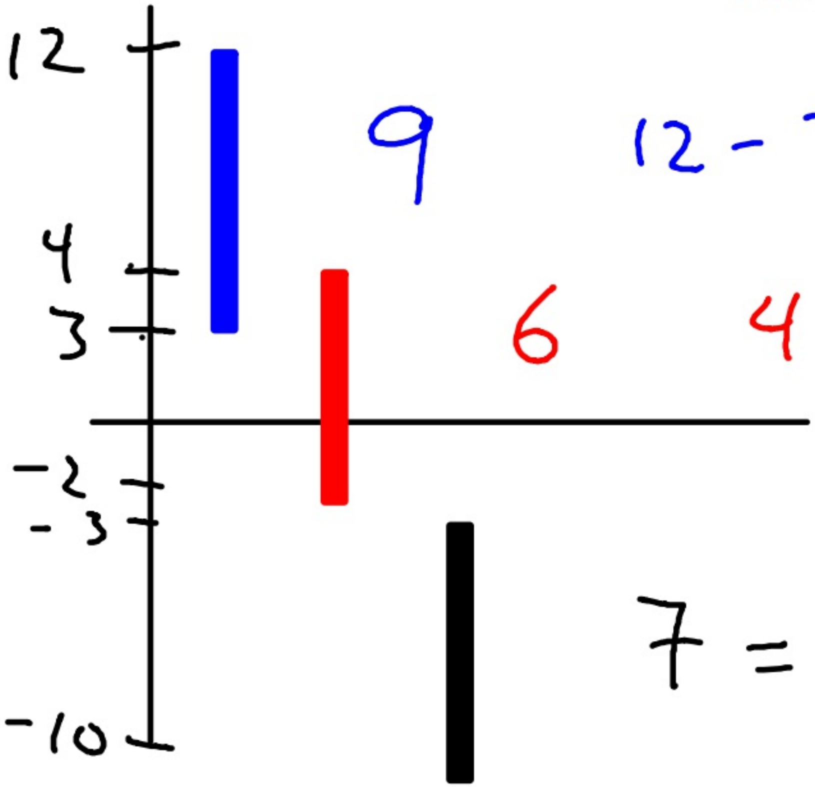
$$\int_a^b \left[\underbrace{f(x)}_{\text{top}} - \underbrace{g(x)}_{\text{bottom}}\right] dx$$

[redacted]



# Vertical Distance

How 'tall' are these rectangles?



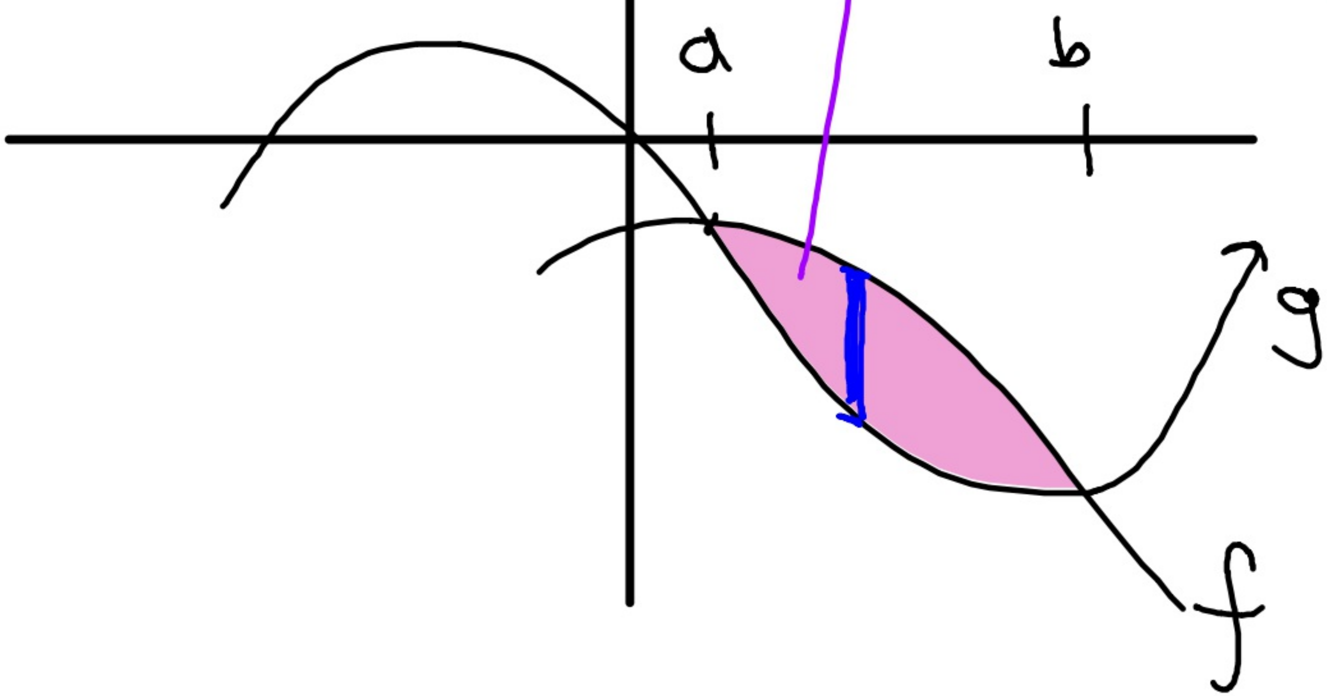

9 $\quad 12 - 3$

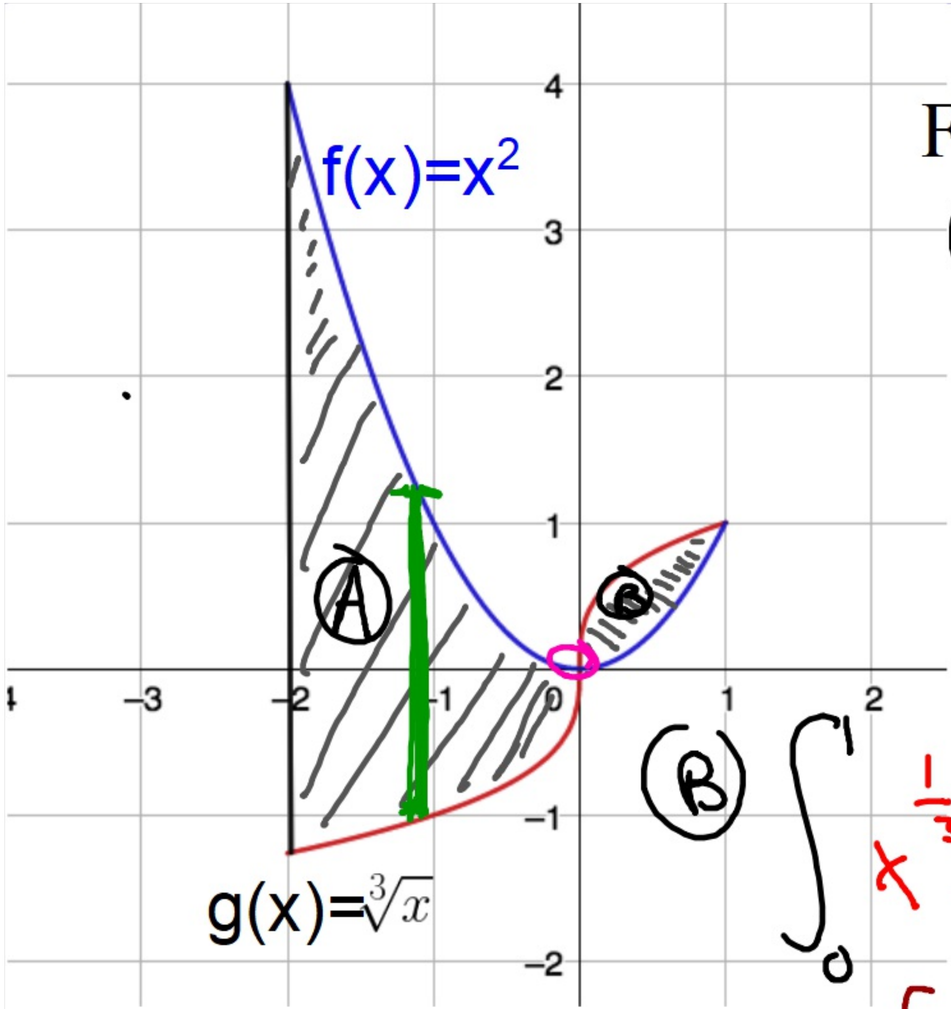
6 $\quad 4 - -2$

$7 = -3 - -10$

# Area Between Curves

$$\int_a^b \underbrace{f(x)-g(x)}_{\text{height of rectangle}} \, dx$$






# Find the area of the shaded region

A $-\int_{-2}^{0}\left(x^2 - x^{\frac{1}{3}}\right) dx$

$$\left[\frac{1}{3}x^3 - \frac{3}{4}x^{\frac{4}{3}}\right]_{-2}^{0}$$

$= 4.557$

B $\int_{0}^{1} x^{\frac{1}{3}} - x^2 \, dx$

$$\left[\frac{3}{4}x^{4/3} - \frac{1}{3}x^3\right]_0^1 = 5/12$$

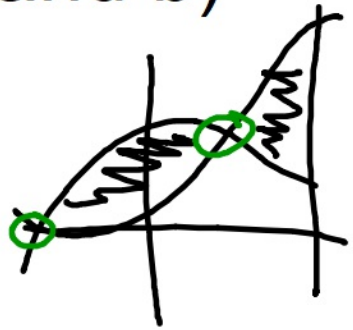
$+$

4.974

Must know the intersection points!!!!

These define the "limits of integration" (the a and b)

Intersection points by hand?



**set functions equal to each other,**

**solve for x by gathering terms on one side,**

**factoring, setting factors = 0**

Intersection points by calc?

**put functions into Y1 and Y2 in calculator**

**use $\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ (calc menu) $\boxed{5}$ INTERSECT**

## How much work to show?

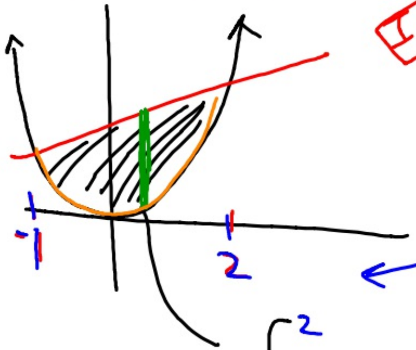
- **take the antiderivative (reverse power rule, etc.)**
  **actually plugging in the upper and lower bounds can be monotonous, so ~~you can~~ evaluate with $\boxed{\text{MATH}}$+9**

$$\int_{\_}^{\_} \underline{\qquad\qquad\qquad}$$

HW #1-6 on handout

BONUS CONTENT (DLC)

Find the area of the region bounded by $y=x^2$ and $y=x+2$



$\boxed{\text{INT?}}$ $x^2 = x+2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x=2, \quad x=-1$

$$\int_{-1}^{2} x+2-x^2 \, dx$$

$$\left[\frac{1}{2}x^2 + 2x - \frac{1}{3}x^3\right]_{-1}^{2} \longrightarrow$$

$$\left(\frac{1}{2}(2)^2 + 2(2) - \frac{1}{3}(2)^3\right) - \left(\frac{1}{2}(-1)^2 + 2(-1) - \frac{1}{3}(-1)^3\right)$$

$$2 + 4 - \frac{8}{3} \quad - \quad \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$6 - \frac{8}{3} \quad - \quad \left(\frac{5}{6} - 2\right)$$

$$\frac{10}{3} \quad - \quad -\frac{7}{6}$$

$$\frac{10}{3} + \frac{7}{6} \to \frac{20}{6} + \frac{7}{6} \to 27/6$$

$\div 3$

$9/2$

Find the area of the region bounded by

$y=x^2$ and $y=\dfrac{2}{x^2+1}$

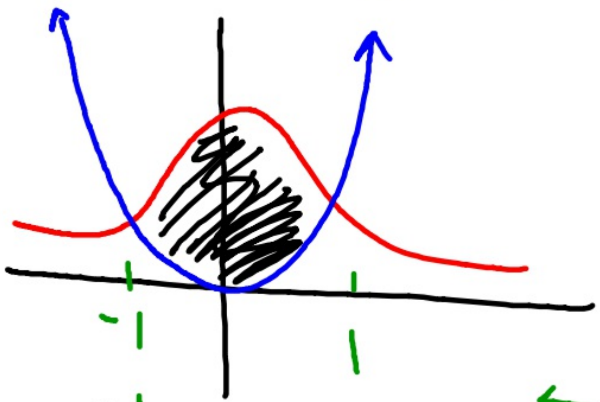
$$\frac{x^2}{1} = \frac{2}{x^2+1}$$

$$2 = x^4 + x^2$$

$$0 = x^4 + x^2 - 2$$

$$0 = (x^2+2)(x^2-1)$$

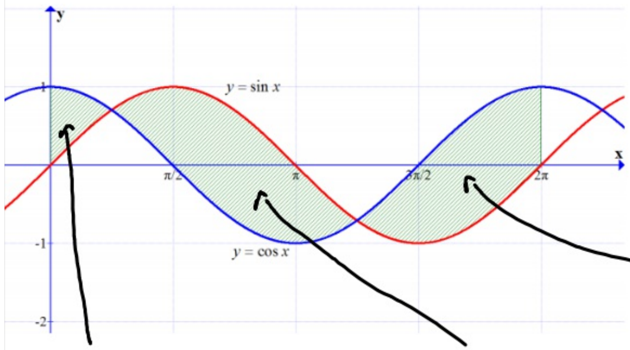
$$\varnothing \qquad x = \pm 1$$




$$\int_{-1}^{1}\left(\frac{2}{x^2+1} - x^2\right)dx$$

use math-9, antideriv. is too hard

$$2\arctan(x) - \frac{1}{3}x^3\Big]_{-1}^{1} = 2.475$$




$$\int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx + \int_{\frac{5\pi}{4}}^{2\pi} (\cos x - \sin x)\,dx$$

$$\sin x + \cos x \Big]_0^{\frac{\pi}{4}}$$

$$\left[\left(\sin\tfrac{\pi}{4} + \cos\tfrac{\pi}{4}\right) - (\sin 0 + \cos 0)\right]$$

$$\sqrt{2} - 1$$

$$-\cos x - \sin x \Big]_{\pi/4}^{5\pi/4}$$

$$\left(-\cos\tfrac{5\pi}{4} - \sin\tfrac{5\pi}{4}\right) - \left(-\cos\tfrac{\pi}{4} - \sin\tfrac{\pi}{4}\right)$$

$$(\sqrt{2}) - (-\sqrt{2})$$

$$2\sqrt{2}$$

$$\sin x + \cos x \Big]_{5\pi/4}^{2\pi}$$

$$(\sin 2\pi + \cos 2\pi) - \left(\sin\tfrac{5\pi}{4} + \cos\tfrac{5\pi}{4}\right)$$

$$1 \quad - \; -\sqrt{2}$$

$$1 + \sqrt{2}$$

$$\sqrt{2} - \cancel{1} + 2\sqrt{2} + \cancel{1} + \sqrt{2}$$

$$4\sqrt{2}$$

Handout #1-~~12~~ 6 on area between curves

-set up integral(s)
-antiderive using FTC2
-use calc for tedious arithmetic ("Math 9")