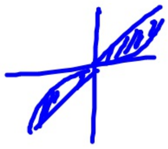
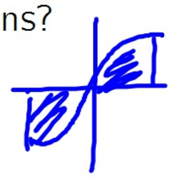
AP Calculus AB DS

HW sols   Due fri   Questions?

1 a. A(1,3) B(3,3)

b. 4/3 or 1.333





2 (intersect at x=-5 and 5) 500/3 or 166.667

3. (intersect at x=0 and 2) 8/3 or 2.667

4. (intersect at x=-2 and 2) 128/3 or 42.667

5. (intersect at 0 and 2) 8/3 or 2.667

6. 128

7. a. A(0,4) B(4,8)

b. 64/3 or 21.333

Reminders:

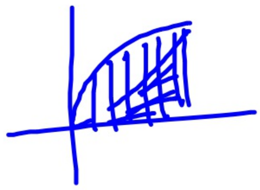
- can reassess tom. and Fri in DS.

- Assess on Area, Definite Integrals on 2/22

- Big honkin' assessment 2/29

$$\int (\text{top} - \text{bottom})\,dx$$

int. points



$$\int_0^3 x^5 - 2x \, dx = \boxed{}$$

$f(x) = g(x)$

solve for x

$$\left[\frac{1}{6}x^6 - x^2\right]_0^3 =$$

Today's Agenda:
- More on the Practice Assessment in DS
- Applying Definite Integrals AP Practice
- Definite Integral Properties
(Will revisit area between 2 curves again in the future)

Practice Assessment

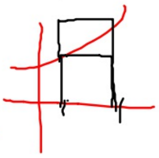
Let's do #1 and 2

Private think time

Turn and Talk

[redacted]

$\int_{1/2}^{3}$ today



I-U1

1. Explain in words why

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$$

in the context of area under a curve. You may use an illustration to accompany your text.

I-U2

2. Find $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$ for f(x) $= -x^2 + 5$ over the interval [0,4]

a b

$-4/3$

$$\int_0^4 -x^2+5\,dx$$

$$\left[-\frac{1}{3}x^3+5x\right]_0^4 = -\frac{1}{3}\cdot 4^3 + 5(4) - 0$$

$-4/3$

I-U1

1. Explain in words why $\lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{f(x_i)\Delta x}_{\text{Area of 1 rectangle}} = \int_a^b f(x)dx$ in the context of area under a curve. You may use an illustration to accompany your text.

[a,b] is where the ∞ rectangles live.

height

base

Infinite rectangles

Sums together "n" rectangles

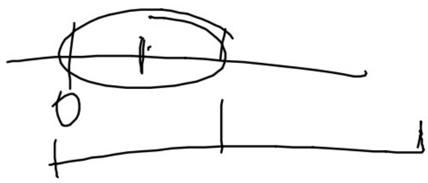
the left side shows a sum of rectangle areas (f(x) height and delta x base) and there are infinitely many of these rectangles. Added together, these give the exact area under the curve.

the right hand side is defined as the area under the curve

I-U2

2. Find $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ for $f(x) = -x^2 + 5$ over the interval [0,4]

How do you define infinity?



the size of a set whose parts can be subdivided
and those subgroups map directly onto the original set

$$\underbrace{1, 2, 3, 4, \ldots}_{\infty}$$

2, 4, 6, 8, ...

$1 \longrightarrow 2$
$2 \longrightarrow 4$
$3 \longrightarrow 6$
$4 \longrightarrow 8$
$5 \longrightarrow 10$
$6 \longrightarrow$
$\vdots$

1	0.34768
2	0.2479995
3 ∞	0.4226512
4	0.875231
6	0.
1	0. ⋮
⋮	

0

#4: MRAM from a function...not a table!

4. Find the midpoint Riemann approximation of $\int_2^6 \frac{3}{x}\,dx$ using 4 intervals of equal width.

$$1\left[f(2.5)+f(3.5)+f(4.5)+f(5.5)\right]$$

$$1.2 + .857 + 0.667 + 0.545$$

$$\boxed{3.269}$$

What is the definite integral?

- a number
- sum of infinite rectangles
- accumulate net change/displacement (??)
- area under the curve

Some key physics concepts to understand:

- Position : distance (cm)

deriv. (slope) antideriv. (area)

- Velocity distance/time (cm/sec)
  - Negative velocity?? (backwards/down)
  - Speed ‖Velocity‖

deriv. (slope) antideriv. (area)

- Acceleration $(cm/s^2$ or $\frac{cm/s}{s})$

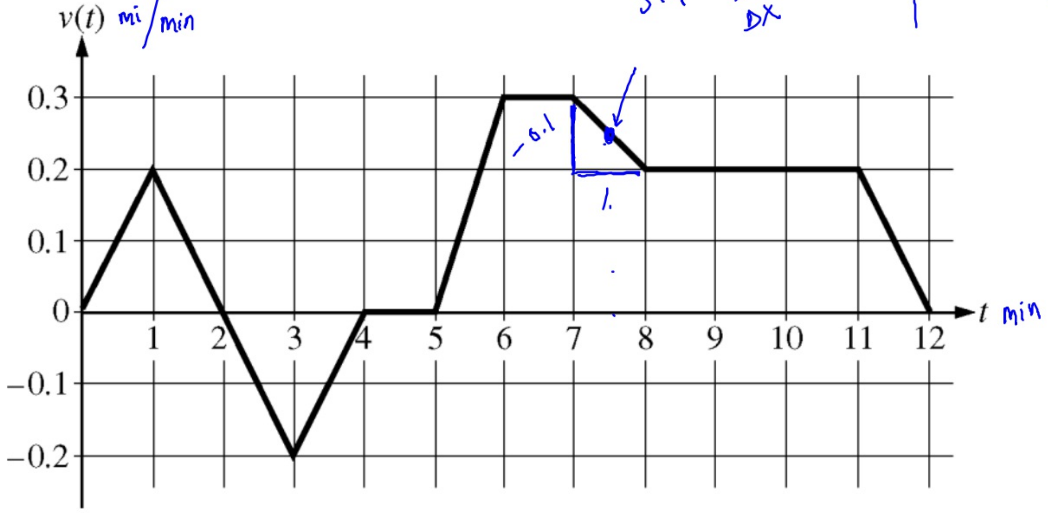
- Displacement

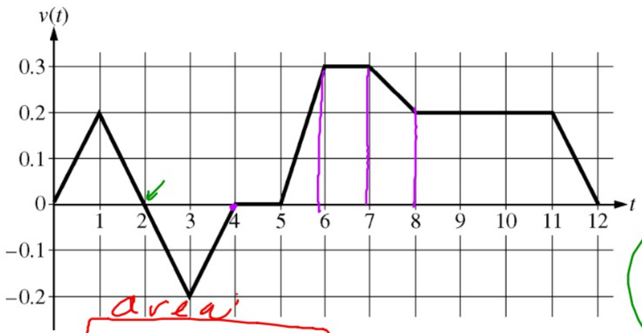
-4 cm/sec

Caren's Bike: 2009AB1: Calculator OK

slope: $\frac{\Delta y}{\Delta x} = \frac{-0.1}{1} = -0.1 \; \frac{mi/min}{min}$

mi/min^2





Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$. Displacement or distance traveled in 12 min. in miles

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer. $t=2$

$\boxed{\text{(d)}}$ Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

$$\overbrace{\int_0^{12} \underbrace{|v(t)|\,dt}_{\text{rectangles}}}^{\text{area}} = 1.8 \text{ mi}$$

Sum

$v(2) = 0$
and velocity after 2
is negative.