

Practice Assessment

Solutions

Ms. M - May 2022

I-U1

- Explain in words why

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_a^b f(x) dx$$

infinite rectangles
 sum of n rectangles' area
 height base exact Area

I-U2

- Find $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$ for $f(x) = -x^2 + 5$ over the interval $[0, 4]$

$-\frac{4}{3}$

$$\text{ew.gr.55.} \Rightarrow \int_0^4 -x^2 + 5 dx = \left[-\frac{1}{3}x^3 + 5x \right]_0^4 = -\frac{1}{3} \cdot 4^3 + 5 \cdot 4 - (0)$$

I-U3a

- Find the left Riemann approximation of $\int_{-4}^{-2} x^2 + x + 1$ using 4 intervals of equal width. Then, determine if this is an over or under approximation and explain how you know.

6.375
over

$$\Delta x = \frac{-2 - (-4)}{4} = \frac{2}{4} = \frac{1}{2}$$

Over: Left approx.
of a decreasing function

$$\text{I-U3b } \frac{1}{2} [f(-4) + f(-3.5) + f(-3) + f(-2.5)] = \frac{1}{2} [5 + 3.625 + 2.5 + 1.625] = 6.375$$

- Find the midpoint Riemann approximation of $\int_2^{6.3} \frac{3}{x} dx$ using 4 intervals of equal width.

3.269

$$\Delta x = \frac{6.3 - 2}{4} = \frac{4.3}{4} = 1$$

$$\text{I-U3c } \int_2^6 \frac{3}{x} dx \approx 1 [f(2.5) + f(3.5) + f(4.5) + f(5.5)] = 1 [1.2 + 0.857 + 0.667 + 0.545]$$

- Shown below are selected values for a differentiable function $f(x)$. Find the difference in the left and right Riemann approximations of $\int_0^8 f(x) dx$ using the intervals indicated by the table.

x	0	2	3	4	5	7	8
$f(x)$	3	4	2	4	2	3	2

Right:

$$2(4) + 1(2) + 1(4) + 1(2)$$

$$+ 2(3) + 1(2)$$

$$\text{Left: } 2(3) + 1(4) + 1(2) + 1(4) + 2(2) + 1(3)$$

$$8 + 2 + 4 + 2 + 6 + 2$$

$$6 + 4 + 2 + 4 + 4 + 3$$

I-A4a

23:

- Find the area of the region bounded by $f(x) = x^5 - 4x^3 + 3x + 4$, the x-axis, and the lines $x = -2$ and $x = 1$.

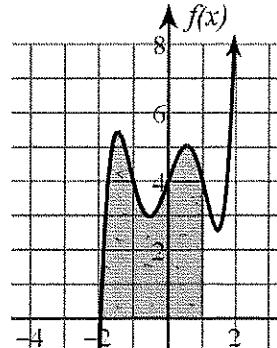
12

$$\int_{-2}^1 x^5 - 4x^3 + 3x + 4 dx$$

$$\left[\frac{1}{6}x^6 - x^4 + \frac{3}{2}x^2 + 4x \right]_{-2}^1$$

$$\left(\frac{1}{6} \cdot 1^6 - 1^4 + \frac{3}{2} \cdot 1^2 + 4 \right) - \left(\frac{1}{6}(-2)^6 - (-2)^4 + \frac{3}{2}(-2)^2 + 4(-2) \right)$$

$$\frac{14}{3} - -\frac{22}{3} = \frac{36}{3} = 12$$



I-A5

$$7. \int_3^4 -\frac{2}{2x-2} dx = -1 \int_3^4 \frac{1}{2x-2} dx = -1 \int_3^4 \frac{1}{2} \frac{1}{x-1} dx = -1 \left[\ln|x-1| \right]_3^4$$

$$-1 [\ln(2(4)-2) - \ln(2(3)-2)]$$

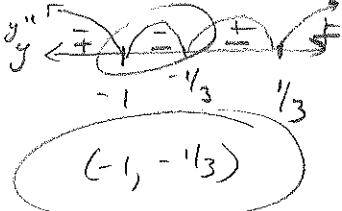
$$-\ln 6 + \ln 4 \Leftarrow -(\ln 6 - \ln 4)$$

D-AD13

8. Find the interval(s) over which $f(x) = \frac{1}{6}(x^3 + x^2 - x)$ is decreasing and concave down.

$$\begin{aligned} y' &= \frac{1}{6}(3x^2 + 2x - 1)' \\ &= \frac{1}{6}(3x-1)(x+1) = 0 \\ &\quad x = \frac{1}{3}, \quad x = -1 \end{aligned}$$

$$\begin{aligned} y'' &= \frac{1}{6}(6x+2) = 0 \\ &\quad x = -\frac{1}{3} \text{ T.P.} \end{aligned}$$



I-A2a

$$9. 2 \int e^{\frac{x}{2}} * \cos(e^{\frac{x}{2}}) dx$$

$$2 \int e^{\frac{1}{2}x} \cdot \cos(e^{\frac{1}{2}x}) dx \Rightarrow 2 \cdot 2 \int \frac{1}{2} e^{\frac{1}{2}x} \cdot \cos(e^{\frac{1}{2}x}) dx$$

want: $\frac{1}{2} e^{\frac{1}{2}x}$

$$4 \int \frac{1}{2} e^{\frac{1}{2}x} \cdot \cos(e^{\frac{1}{2}x}) dx$$

$$4 \cdot \sin(e^{\frac{1}{2}x}) + C$$

I-A1b

$$10. \int \frac{8x \cos 4x^2}{\sin 4x^2} dx$$

$$\int 8x \cdot \cos 4x^2 \cdot \frac{1}{\sin 4x^2} dx$$

$\checkmark 11$

$$\boxed{\ln |\sin 4x^2| + C}$$

I-A2b

$$11. \int 5x \sqrt{3x+1} dx \quad \int 5x \cdot (3x+1)^{1/2} du$$

$$\text{Let } u = 3x+1 \Rightarrow \frac{du}{dx} = 3 \Rightarrow \frac{du}{3} = dx$$

$$\frac{u-1}{3} = x$$

$$\int 5 \cdot \frac{u-1}{3} \cdot u^{1/2} \frac{du}{3}$$

- A) $\frac{2}{9}(3x+1)^{\frac{5}{2}} - \frac{10}{27}(3x+1)^{\frac{3}{2}} + C$
 B) $\frac{1}{21}(3x+1)^{\frac{7}{3}} - \frac{1}{12}(3x+1)^{\frac{4}{3}} + C$
 C) $\frac{2}{21}(3x+1)^{\frac{7}{3}} - \frac{1}{6}(3x+1)^{\frac{4}{3}} + C$
 D) $\frac{8}{45}(3x+1)^{\frac{5}{2}} - \frac{8}{27}(3x+1)^{\frac{3}{2}} + C$

$$\int \frac{5}{9}(u-1)u^{1/2} du$$

$$\int \frac{5}{9}(u^{3/2} - u^{1/2}) du$$

$$\frac{2}{9}u^{5/2} - \frac{10}{27}u^{3/2} + C \quad \leftarrow \frac{5}{9} \left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} \right) + C$$

A

$$y = 2x^2 + 3x - 2$$

I-A3

$$12. \text{Find } y \text{ if } \frac{dy}{dx} = 4x+3 \text{ and } y(-2)=0. \quad y = 2x^2 + 3x + C$$

$$0 = 2(-2)^2 + 3(-2) + C \Rightarrow 0 = 8 - 6 + C \Rightarrow C = -2$$

$$\int dy = \int 4x+3 dx$$

(-2, 0)

$$\boxed{y = 2x^2 + 3x - 2}$$