2 kinds of Integrals

Indefinite Integral:

$$\int f(x)\,dx$$

yields

a family of functions

Definite Integral.

$$\int_a^b f(x)\,dx$$

limits of Integration

yield

a number.

Example:

[Def. Int:] • $\int_3^4 2x \cdot dx = \left[x^2 \right]_3^4$

$4^2 - 3^2$

$16 - 9$

7

• $\int_7^8 3x^2 \cdot dx$

$\left[x^3 \right]_7^8$

$8^3 - 7^3$

$512 - 343 = 169$

Summation Notation summ-ary

Write as a sum using sigma notation.

1+2+3+4+5+6

Stopping (size)

$$\sum_{i=1}^{6} i$$

formula for turning i into the seq. terms.

index (Starting #)

Write as a sum using sigma notation:

5+10+15+20+25+30

$$\sum_{i=1}^{6}$$

$\underbrace{5(1+2+3+4+5+6)}$

5(1)+5(2)+5(3)+5(4)+5(5)+5(6)

$$5\sum_{i=1}^{6} i \Longleftrightarrow \sum_{i=1}^{6} 5i$$

[redacted]

[redacted]

Write as a sum using sigma notation:

$5 + 10 + 15 + 20 + 25 + 30 + \ldots + 5n$

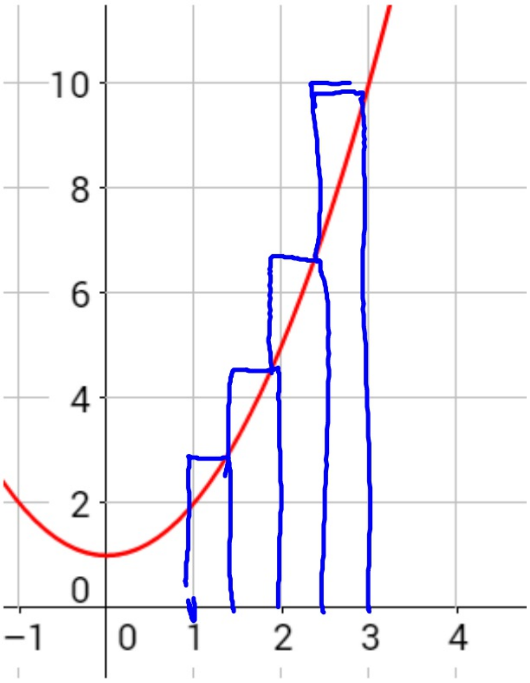
$$\sum_{i=1}^{n} 5i$$

Write as a sum using sigma notation:

5 + 10 + 15 + 20 + 25 + 30 + ...

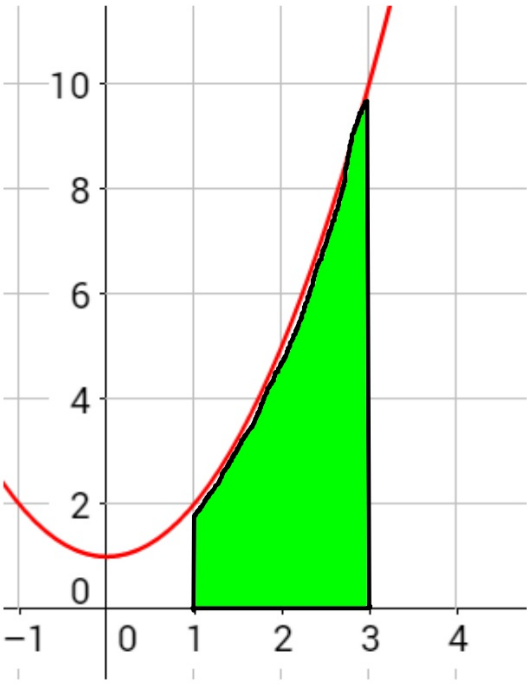
$$\lim_{x \to 3} 5x = 15$$

$$\sum_{i=1}^{\infty} 5i \Longleftrightarrow \lim_{n\to\infty} \sum_{i=1}^{n} 5i$$



How do you find the area under this curve from x = 1 to x =3?

Think of at least two different ways, then tell your neighbors.



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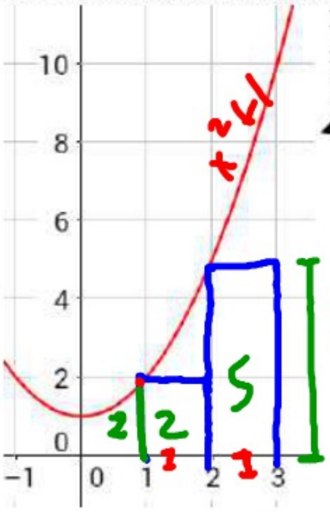
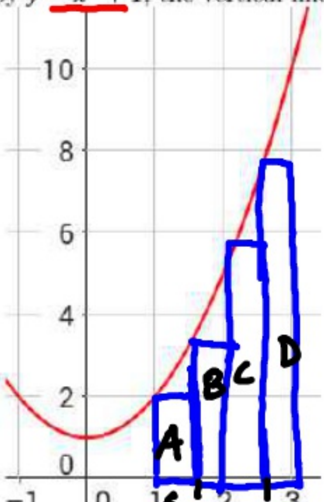
How to find the area under a curve?



Find a method to approximate the area enclosed by $y = x^2 + 1$, the vertical lines x=1 and x=3, and the x-axis.
(This is called 'the area under the curve')



First approximation 7

How can you improve your method?

Second approximation

$1\cdot 2 + 1\cdot 5 =$
b h b h

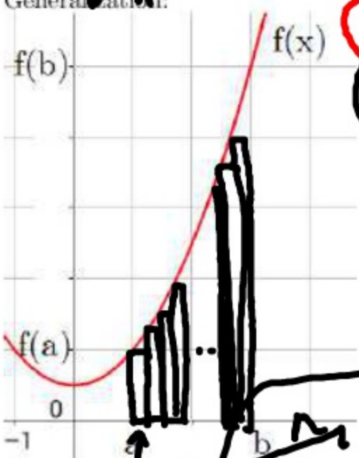
$\Delta x = \frac{3-1}{4} = \frac{1}{2}$

$\Delta x = \frac{b-a}{n}$ width

\# of intervals

$\overbrace{\frac{1}{2}\cdot 2}^{A} + \overbrace{\frac{1}{2}\cdot \underbrace{3.25}_{f(1.5)}}^{B} + \overbrace{\frac{1}{2}\cdot 5}^{C} + \underbrace{\frac{1}{2}\cdot 7.25}_{D}$

Generalization:



$\Delta x = \frac{b-a}{n}$

$\sum_{i=1}^{n} f(x_i)\cdot\Delta x$ — $\boxed{8.75}$

$\int f(x)\, dx$

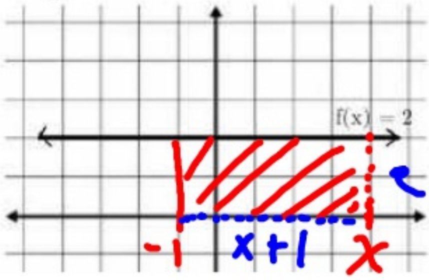
Riemann definition of Definite Integral: if f is a continuous function on [a,b] and this interval is equally divided into n intervals of width $\Delta x = \frac{b-a}{n}$, and if $x_i = a + i\Delta x$ is the right endpoint of subinterval i, then:

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$$

Connection between Area and Antiderivatives and Slope

For each function, use geometry to find the area A(x) *under the function* f(x) *between* -1 and some arbitrary point x (or, over the interval [-1,x]). *Then, find* A'(x). What do you notice about f(x) and A'(x)?

1.

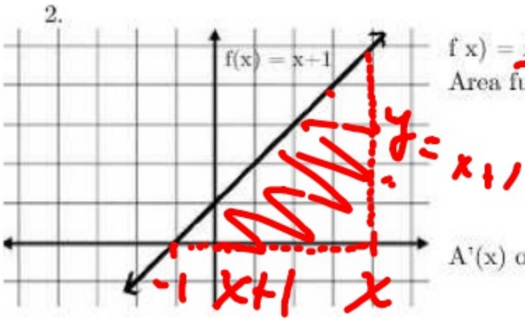


f(x) = 2

Area function A(x) = $2(x+1) = 2x+2$

A'(x) or $\frac{dA}{dx}$ = 2

2.



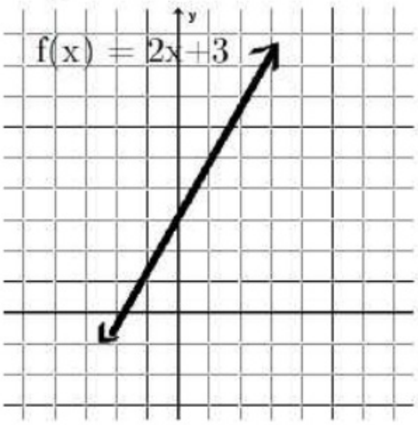
f x) = x+1

Area function A(x) = $\frac{1}{2}(x+1)(x+1)$

$\frac{1}{2}(x^2+2x+2) = \frac{1}{2}x^2+x+1$

A'(x) or $\frac{dA}{dx}$ =

3.



f (x) = 2x+3

Area function A(x) =

A'(x) or $\frac{dA}{dx}$ =

Now go back and find the area under the curve using the FTC:

1. $\int_{-1}^{x} 2\,dx$

2. $\int_{-1}^{x} x+1\,dx$

3. $\int_{-1}^{x} 2x+3\,dx$