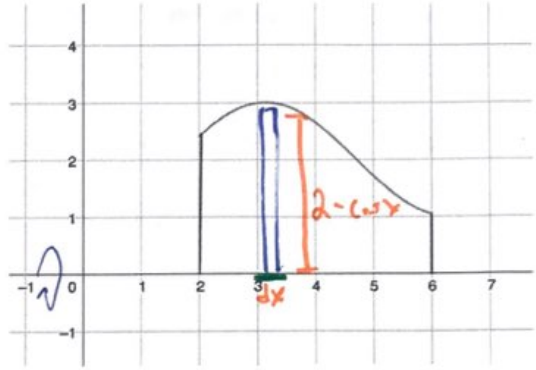
1. Find the volume of the solid generated by revolving the region bounded by $f(x) = 2 - \cos x$ and the vertical lines $x = 2$ and $x = 6$ about the x-axis. ~~Show all wo~~rk.

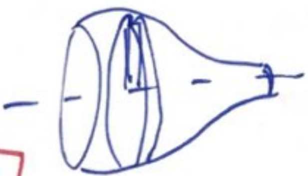


• Vol of 1 cylinder: $\pi r^2 \cdot h$

$V = \pi (2 - \cos x)^2 \cdot dx$

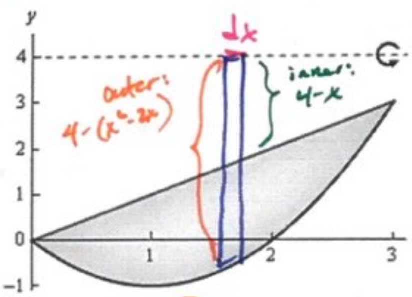
• Vol of all cylinders

$$V = \int_2^6 \pi (2 - \cos x)^2 dx \Rightarrow \pi \int_2^6 (2 - \cos x)^2 dx$$



Calc. $\int$ $\boxed{22.810\pi \approx 71.659\ u^3}$

2. Set-up a single integral to calculate the volume of the solid generated when the region bounded by $f(x) = x^2 - 2x$ and $g(x) = x$ is revolved around the axis $y = 4$. Then use a calculator to find that volume.



$$V = \pi \int_0^3 (4 - (x^2 - 2x))^2 - (4 - x)^2\, dx$$

$$V = \pi \cdot \int_0^3 (4 - x^2 + 2x)^2 - (4 - x)^2\, dx$$

$(\pi R^2 - \pi r^2)\, dx$

$\boxed{30.6\pi \approx 96.133\ u^3 \quad \frac{153\pi}{5}}$

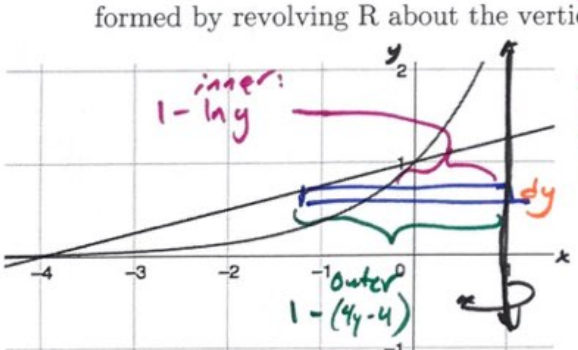
I-A5b

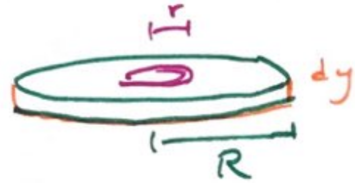
3. Let R be the region bounded by $g(x) = e^x$ and $h(x) = \frac{1}{4}x + 1$. Find the volume of the solid formed by revolving R about the vertical line $x = 1$. (Calc ok)

$y = e^x$ $y = \frac{1}{4}x + 1$

$\ln y = \ln e^x$ $y - 1 = \frac{1}{4}x$

$\ln y = x$ $4y - 4 = x$





$$V = \pi \int_0^1 (1 - (4y - 4))^2 - (1 - \ln y)^2\, dy$$

$$V = \pi \int_0^1 (1 - 4y + 4)^2 - (1 - \ln y)^2\, dy$$

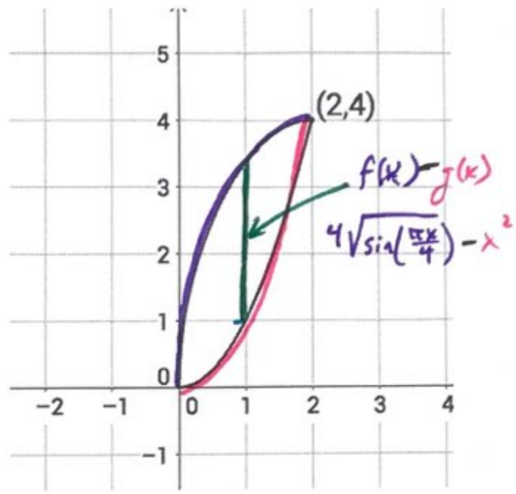
$$V = \pi \int_0^1 (5 - 4y)^2 - (1 - \ln y)^2\, dy$$

$\boxed{5.333\pi \approx 16.755\ u^3}$

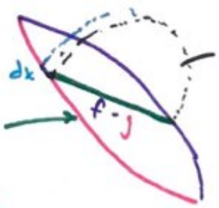
I-A5c

Let R be the first-quadrant region enclosed by

$f(x) = 4\sqrt{\sin(\frac{\pi x}{4})}$ and $g(x) = x^2$.



4. Let R be the base of a solid whose cross-sections perpendicular to the x-axis are semicircles. Find the volume of this solid. (Calc ok)



this is diameter!

So $r = \frac{f-g}{2}$

Area? $A = \frac{1}{2}\pi r^2$

$$A = \frac{\pi}{2}\left(\frac{f-g}{2}\right)^2$$

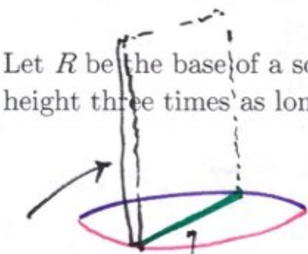
$$A = \frac{\pi}{2} \cdot \frac{(f-g)^2}{4}$$

$$A = \frac{\pi}{8}(f-g)^2$$

$$V = \int_0^2 \frac{\pi}{8}(f-g)^2\,dx \Rightarrow \frac{\pi}{8}\int_0^2 \left(4\sqrt{\sin(\tfrac{\pi x}{4})} - x^2\right)^2 dx$$

$\boxed{2.676\ u^3}$

5. Let R be the base of a solid whose cross-sections perpendicular to the x-axis are rectangles with height three times as long as the base. Find the volume of this solid. (Calc ok)



3(f-g)

f-g

Area? $A = \underbrace{(f-g)}_{\text{base}} \cdot \underbrace{3(f-g)}_{\text{height}}$

$$A = 3(f-g)^2$$

$$V = \int_0^2 3(f-g)^2\,dx = 3\int_0^2 \left(4\sqrt{\sin(\tfrac{\pi x}{4})} - x^2\right)^2 dx$$

$\boxed{20.441\ u^3}$

I-A7a

10. Find the average value of $f(x) = \frac{1}{x}$ over the interval [1,3]

$$\boxed{f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

$$\frac{1}{3-1}\int_1^3 \frac{1}{x}\,dx$$

$$\frac{1}{2}\left[\ln|x|\right]_1^3 \rightarrow \frac{1}{2}\left[\ln|3| - \ln|1|\right]$$

$$\frac{1}{2}\left[\ln 3 - 0\right] \rightarrow \frac{1}{2}\ln 3 \rightarrow \ln\sqrt{3}$$

11. Let $Q'(t) = 1 - \cos(\frac{\pi t}{5})$ model the rate, in hundreds of people per hour, enter an amusement park. Using correct units, explain the meaning of $\frac{1}{5}\int_2^7 Q'(t)\,dt$ in context. Then, find its value.

average rate, in hundreds of people per hour, of people entering park over hours t = 2 to t = 7.

$$= \frac{1}{5}\underbrace{\int_2^7 \left(1 - \cos\left(\frac{\pi}{5}t\right)\right)dt}_{\text{Math - 9}}$$

$$= \frac{1}{5}(8.027)$$

$$= 1.605 \rightarrow 160 \text{ ppl/hr}$$