Good afternoon: no warm up, check hw answers (they've been online) then we'll randomize and learn about applying definite integrals

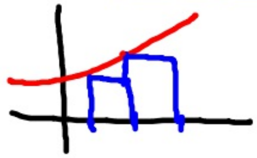
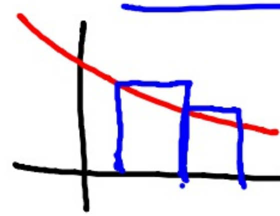
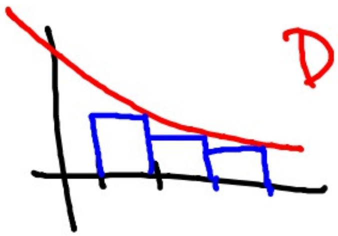
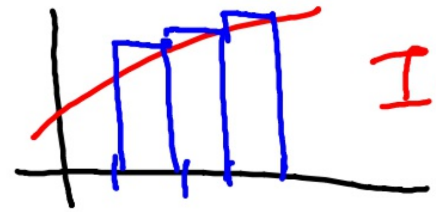
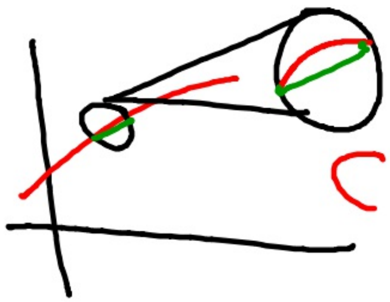
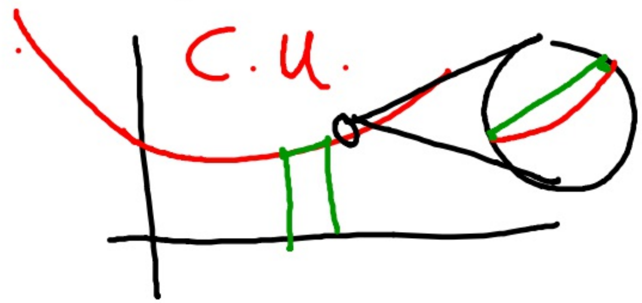
1. LRAM 63, RRAM 58
2. LRAM 69, RRAM 55
3. LRAM 39, RRAM 48
4. LRAM 61, RRAM 65
5. 16.5
6. 17.5
7. 1.575 $\left(\frac{496}{315}\right)$
8. 3.269 $\left(\frac{3376}{1155}\right)$
9. 26
10. 4.467 $\left(\frac{67}{15}\right)$
11. 22
12. 3.367 $\left(\frac{101}{30}\right)$

Test Monday
same 3 skills
as last test

Visibly random grouping

Wrapping up Riemann Sums for Approximating Integrals

Determining over- vs underestimate with integral approximation methods

	UNDER		OVER	
LRAM		INC.		DEC
RRAM		DEC		INC.
TRAPEZ.		C.D.		C.U.

[redacted]

MRAM from a table

Approximate $\int_{6}^{24} f(x)dx$ using 3 midpoint rectangles as indicated by the table.

x	6	9	12	15	18	21	24
$f(x)$	1.5	7	12	10	8	14	19

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{18}{3}$

$\Delta x = 6$

$f(x_i)\Delta x$

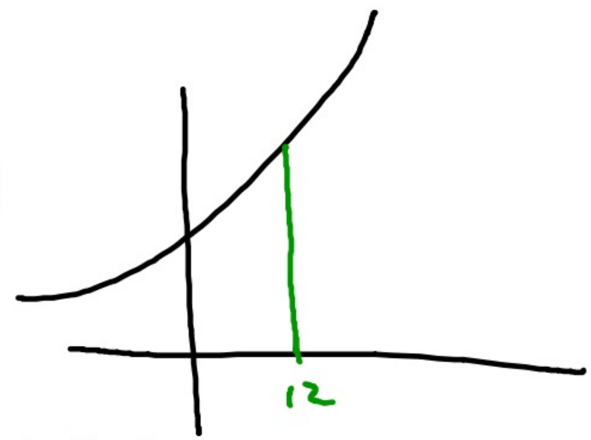
$7 \cdot (6) + 10(6) + 14(6)$

$42 + 60 + 84$

186

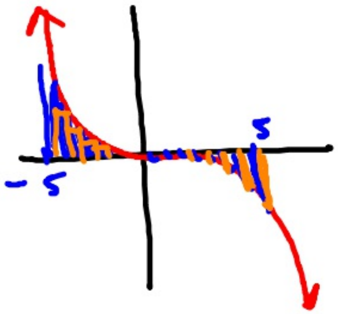
A riddle:

$$\int_{12}^{12} 4e^{4x}\,dx = 0$$



What is the exact answer...mentally?

A riddle:

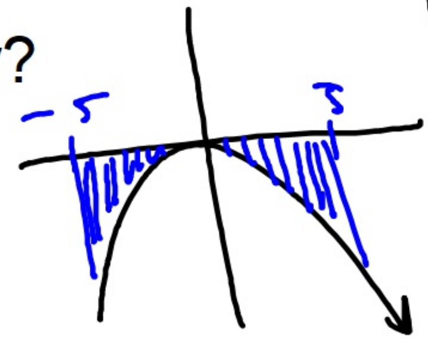


$$\int_{-5}^{5} -x^3\,dx = 0$$

odd function

$$\int_{-5}^{5} -x^2\,dx \neq 0$$

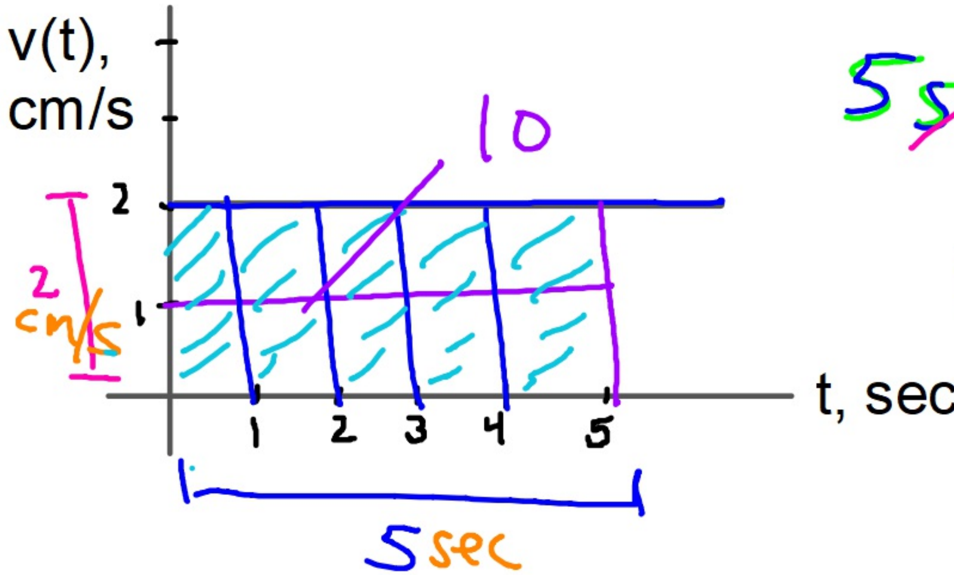
What is the exact answer...mentally?



Integration as Accumulation

A snail travels 2 cm/s along a stick. After 5 seconds, how far has it traveled?

$v(t) = 2$ $[0, 5]$



$$\boxed{5\,\cancel{\text{sec}} \times \frac{2\text{ cm}}{\cancel{\text{sec}}} = 10\text{ cm}}$$

Note that we can't say *where* the snail is without more information

We just know he has been displaced 10 cm.

$v(t) = 2 \text{ cm/sec}$

pt. 0

If its starting position was 6 cm from a ladybug, what is its position at t=8?

Displacement: 16 cm

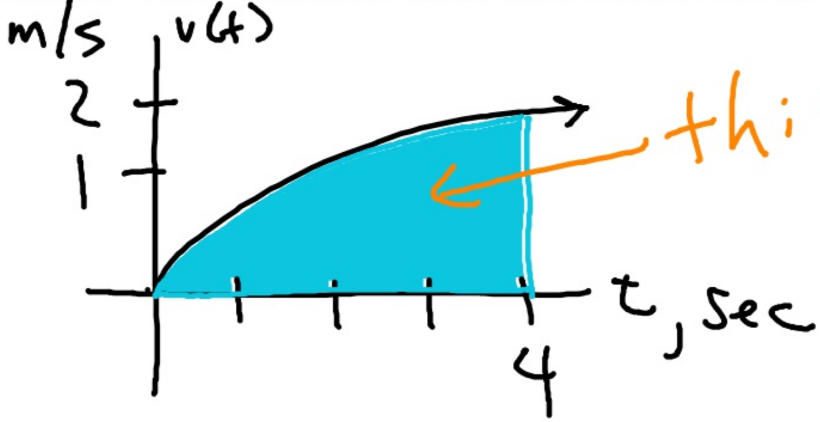
$x(0) = 6$

$x(8) = 6 + 16$

22 cm

$$x(t) = x_0 + \int_0^t f(z)\,dz$$

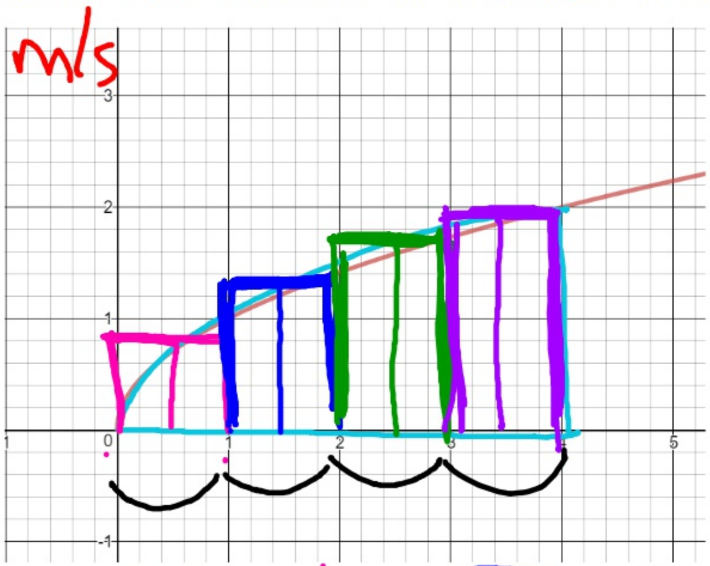
If an object moving in a straight line has velocity $v(t)=\sqrt{t}$ m/s, how far does it travel in the first 4 seconds?



this area is how far the object moves in 4 sec.

$$\text{Area} = \int_0^4 \sqrt{t} \cdot dt$$

If an object moving in a straight line has velocity $v(t)=\sqrt{t}$ m/s, how far does it travel in the first 4 seconds?



$$\int_0^4 \sqrt{t}\,dt \approx 5.384$$

meters

[redacted]

Approximate this integral using 4 midpoint rectangles.

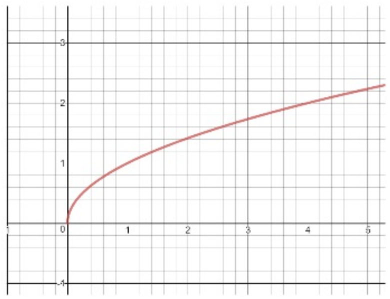
$$\Delta x = \frac{4-0}{4} = 1$$

$$\approx \underbrace{\sqrt{0.5}}_{m/\cancel{s}} \times \cancel{1} + \sqrt{1.5} \times 1 + \sqrt{2.5} \times 1 + \sqrt{3.5} \times 1 = 5.384$$

[redacted]

Do we <u>really</u> need to approximate??

If an object moving in a straight line has velocity $v(t)=\sqrt{t}$ m/s, how far does it travel in the first 4 seconds?



$$\int_0^4 \sqrt{t}\,dt$$

$v(t) = \sqrt{t}$

$=$

position at 4 $x(4)$
minus position at 0
$x(0)$

$$\frac{dx}{dt} = t^{1/2}$$

$$\cancel{\int} dx = \int t^{1/2} \cdot dt$$

$$x(t) = \frac{2}{3} t^{3/2} + C$$

$$x(4) = \frac{2}{3}\cancel{(4)^{3/2}}\,8 + C$$

$$x(4) = \frac{16}{3} + C \qquad x(0) = C$$

$x(4) - x(0)$

$$\frac{16}{3} + \cancel{C - C} \longrightarrow \frac{16}{3} = 5.\overline{3}$$

meters

[redacted]

We just did something very profound

$$\int_0^4 \sqrt{t}\,dt = X(4) - X(0)$$

$$v(t) = \frac{dx}{dt}$$

The Fundamental Theorem of Calculus, Part II

Suppose F(x) is an antiderivative of f(x).
Then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

F is antiderivative of f.

$\therefore F' = f$

(therefore)

*not yet proven! we will need part 1 for that

Suppose F(x) is an antiderivative of f(x).
Then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

How to evaluate a definite integral:

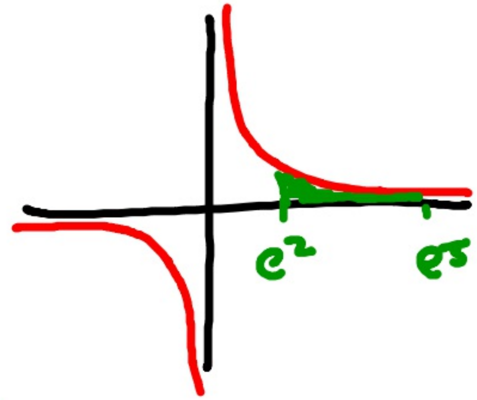
1 Find the antiderivative of the integrand (leave off the +C)
2 Plug in b and plug in a
3 Subtract F(b)-F(a)
4 yay thats it

An example:

$$\int_{e^2}^{e^5} \frac{2}{x}\,dx$$

$$= 2\ln|x|\Big]_{e^2}^{e^5} = 2\underbrace{\ln e^5} - 2\underbrace{\ln e^2}$$

$$10 - 4 = 6$$



Another:

$$\int_{-3}^{2} \frac{1}{5}x^3 - \frac{1}{10}x^2 \cdot dx$$

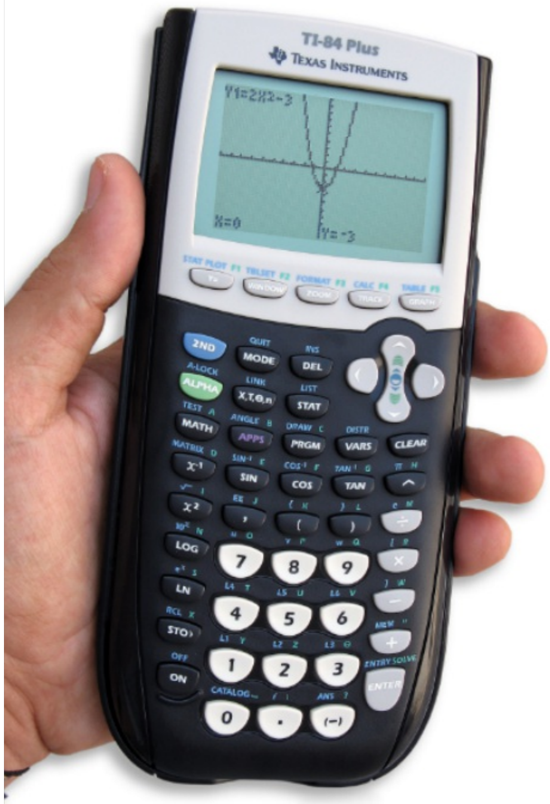
$$\frac{1}{20}x^4 - \frac{1}{30}x^3 \Big]_{-3}^{2} = \left(\frac{1}{20}(2^4) - \frac{1}{30}(2)^3\right)$$

$$-\left(\frac{1}{20}(-3)^4 - \frac{1}{30}(-3)^3\right)$$

$$\left(\frac{4}{5} - \frac{8}{30}\right) - \left(\frac{81}{20} + \frac{27}{30}\right)$$

$$\frac{16}{20} - \frac{8}{30} - \frac{81}{20} - \frac{27}{30}$$

$$\frac{-65}{20} - \frac{35}{30} \longrightarrow \text{yes.}$$



Checking answer

$\boxed{\text{MATH}}$

9

$$\int_{-5}^{5} -x^3 dx$$

We now know how to take *definite integrals*
and not just approximate them, but to find the exact answer

$$\int \ln(x)\,dx$$

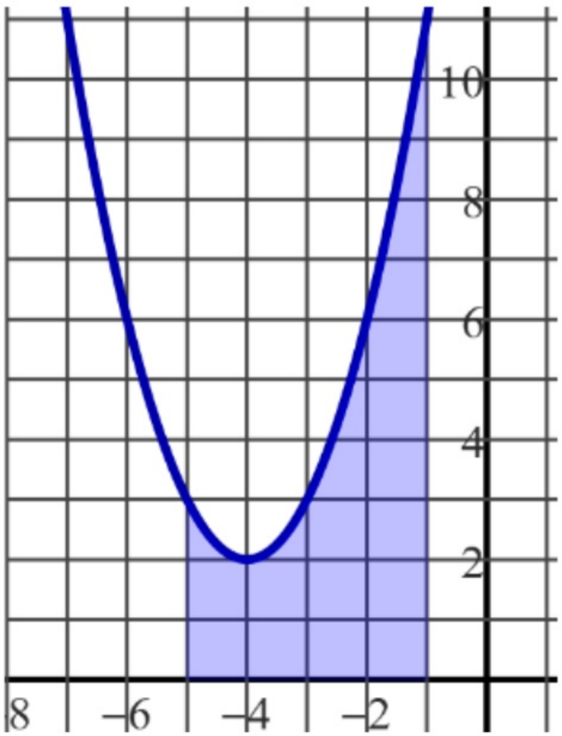
Meaning we can now find <u>exact</u> areas of funky spaces

ex $\int_{-3}^{5} x^2+2\,dx$ → Riemam Sums $\approx$

→ FTC(2) $=$

Find the area of the region

$y = x^2 + 8x + 18$;



some dang review!!!

Convert each representation to a definite integral or an infinite Riemann sum as appropriate.

$$\int_2^4 \frac{1}{x+3}\,dx$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{2+\frac{2}{n}i+3} \cdot \frac{2}{n}$$

$\Delta x = \frac{2}{n}$

$$\underbrace{2+\tfrac{2}{n}i}_{\text{"}x\text{"}}$$

$a + \Delta x_i\, i$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{\frac{3k}{n}}\ \frac{3}{n}$$

$$\int_0^3 \sqrt{x}\,dx$$

$$\int_1^6 2x \sin x^2\,dx$$

HW

p. 288 #5-20