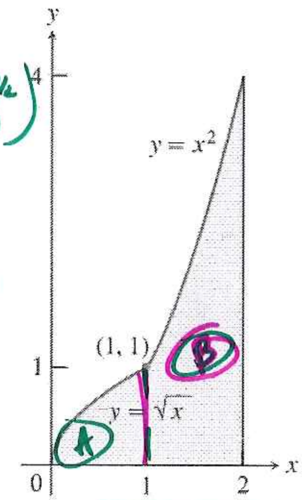
Practice Assessment Q3 #3

I-A4a NO CALC

1. Find the exact area of the shaded region. Show all work.



A $\int_0^1 \sqrt{x}\,dx = \int_0^1 x^{1/2}\,dx = \frac{2}{3}x^{3/2}\Big]_0^1 = \left(\frac{2}{3}(1)^{3/2}\right) - \left(\frac{2}{3}(0)^{3/2}\right)$

$\frac{2}{3} - 0$

$= \frac{2}{3}$ A

B $\int_1^2 x^2\,dx = \frac{1}{3}x^3\Big|_1^2 =$

$\left(\frac{1}{3}(2)^3\right) - \left(\frac{1}{3}(1)^3\right)$

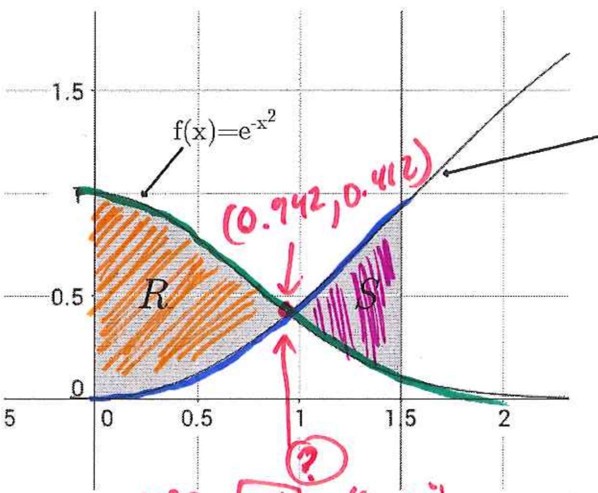
$\frac{8}{3} - \frac{1}{3} = \frac{7}{3}$ B

A + B

$\frac{2}{3} + \frac{7}{3} = \frac{9}{3} = \boxed{3\ u^2}$

I-A4b CALC OK

2. Let $f(x) = e^{-x^2}$ and $g(x) = 1 - \cos(x)$. The regions R and S are bounded by $f(x), g(x)$, the y-axis, and the vertical line $x = 1.5$. Find the total shaded area. Show the setup of your integrals and all related calculations.



use 2nd → "CALC" → INTERSECT
TRACE

$\int_a^b (\text{top} - \text{bottom})\,dx$

R: $\int_0^{0.942} e^{-x^2} - (1 - \cos x)\,dx$

$\approx 0.591\ u^2$

MATH

CALC 9

S: $\int_{0.942}^{1.5} 1 - \cos x - e^{-x^2}\,dx$

$= 0.237\ u^2$

MATH - 9

$\boxed{\text{TOTAL AREA: } = 0.828\ u^2}$

I-U4: NO CALC

Let $f(x) = \int_3^{2x} 2t^2 - 3t - 2\,dt$.

use the FTC

3. Find $f'(x)$. Simplify your answer.

$f'(x) = [2(2x)^2 - 3(2x) - 2] \cdot 2$ ← Chain Rule

$f'(x) = [8x^2 - 6x - 2] \cdot 2 \rightarrow \boxed{16x^2 - 12x - 4 = f'(x)}$

4. Find all intervals where $f(x)$ is increasing. Justify your answer.

f increasing! f' needs to be positive...

See pg 2

④ f increasing? f' needs to be positive...

From #3: $f'(x) = 16x^2 - 12x - 4$

FIND C.N.

Set $f'(x) = 0$

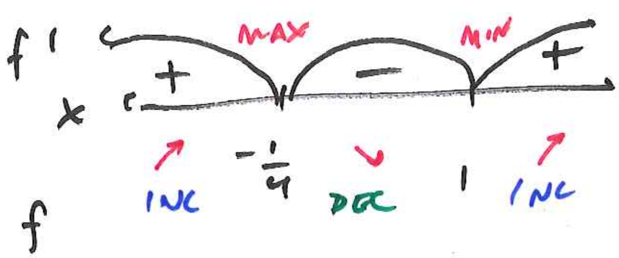
$$16x^2 - 12x - 4 = 0$$

$$4(4x^2 - 3x - 1) = 0$$

$$4(4x + 1)(x - 1) = 0$$

$4x + 1 = 0$ $x = 1$

$x = -1/4$ C.N.



$f'(0) = -$

$f'(1000) = +$

$f'(-1\ldots) =$

f is increasing on $(-\infty, -\frac{1}{4})$ and $(1, \infty)$

b/c $f'(x)$ is positive there.

I-U7 NO CALC

Suppose $f(x)$ and $h(x)$ are continuous functions such that $\int_1^9 f(x)\,dx = -1$, $\int_7^9 f(x)\,dx = 5$, $\int_7^9 h(x)\,dx = 4$.

5. $\int_9^7 [h(x) - f(x)]\,dx$ $= -\int_7^9 h - f$

$-\left[\int_7^9 h - \int_7^9 f\right] \to -[4-5] \to --1 = 1$

6. $\int_1^7 f(x)\,dx$ $= \int_1^9 f + \int_9^7 f$

$\int_1^9 f - \int_7^9 f \to -1-4 \to -5$

I-U5 NO CALC

7. $\int_4^9 2x - \frac{1}{\sqrt{x}}\,dx$ $= \int_4^9 2x - x^{-1/2}\,dx \to x^2 - 2x^{\frac{1}{2}}\Big]_4^9 \to x^2 - 2\sqrt{x}\Big]_4^9$

$\frac{1}{x^{1/2}}$

$(9^2 - 2\sqrt{9}) - (4^2 - 2\sqrt{4})$

$81 - 6 \quad - (16-4)$

$75 - 12 = 63$

8. If $\int_{-2}^{2}(x^3 + k)\,dx = 16$, then what is the value of k?

see pg. 4

$\boxed{k=4}$

I-A3 NO CALC

9. Suppose $f'(x) = 2\sqrt{x}$ and $f(1) = 4$. Find the value of $f(4)$.

See pg 4

$\frac{40}{3}$

I-U3b CALC OK

RADIAN MODE!

10. Find the midpoint rectangle approximation for $\int_3^7 \underbrace{\tan(0.2x)}_{f}\,dx$ using 4 rectangles of equal width. [3 decimal places of accuracy.]

$\Delta x = \frac{7-3}{4} = 1$

3 - 4 - 5 - 6 - 7

x	3.5	4.5	5.5	6.5
f	.842	1.260	1.965	3.602

$\int_3^7 \tan(0.2x)\,dx \approx$

$= 1(.842 + 1.260 + 1.965 + 3.602)$

Δx

$\boxed{\approx 7.901}$

I-U3c NO CALC

11. An awesome rocket ship is in the air and doing cool rocket things. Its velocity $v(t)$ is a differentiable, strictly increasing function. Selected values are given below. Using correct units, explain the meaning of $\int_2^{10} v(t)\,dt$ in the context of this problem. Then, approximate the value of $\int_2^{10} v(t)\,dt$ using the 4 trapezoids indicated by the table.

t	2	4	6	8	10
$v(t)$, m/s	12	18	27	38	52

See pg 5

(8) If $\int_{-2}^{2}(x^3+k)\,dx = 16$, then what is the value of k?

FTC 2

$$\frac{1}{4}x^4 + kx\Big]_{-2}^{2}$$

$$\left(\underbrace{\tfrac{1}{4}(2)^4}_{16} + k\cdot 2\right) - \left(\underbrace{\tfrac{1}{4}(-2)^4}_{16} + k(-2)\right)$$

$$4 + 2k \quad - (4 - 2k)$$

$$\cancel{4} + 2k - \cancel{4} + 2k = 16$$

$$4k = 16 \longrightarrow \boxed{k = 4}$$

(9) Suppose $f'(x) = 2\sqrt{x}$, $f(1) = 4$. $f(4) = ?$

(1,4)

✓Integrate to get f

$$f = \int 2\sqrt{x}\,dx$$

$$f = \int 2x^{1/2}\,dx$$

$$f(x) = 2\cdot\frac{x^{3/2}}{3/2} + C$$

$$f(x) = 2\cdot\frac{2}{3}x^{3/2} + C$$

$$f(x)\ \frac{4}{3}x^{3/2} + C \longrightarrow$$

$$f(1) = 4 = \frac{4}{3}(1)^{3/2} + C$$

$$4 = \frac{4}{3} + C$$

$$4 - \frac{4}{3} = C \longrightarrow \frac{12}{3} - \frac{4}{3} = C$$

$$\frac{8}{3} = C$$

$$f(x) = \frac{4}{3}x^{3/2} + \frac{8}{3}$$

$$f(4) = \frac{4}{3}(4)^{3/2} + \frac{8}{3}$$

$$\frac{4}{3}(4^{1/2})^3 + \frac{8}{3}$$

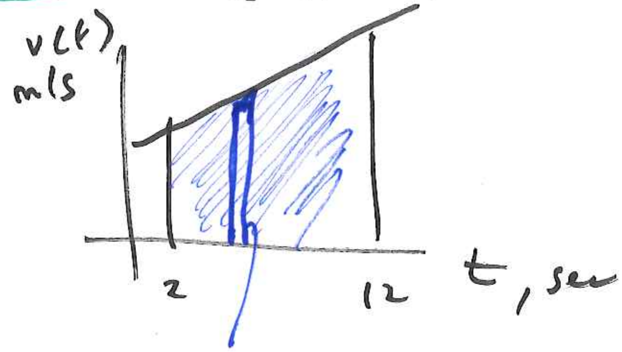
$$\frac{32}{3} + \frac{8}{3} \longrightarrow \boxed{\frac{40}{3}}$$

I-U3c NO CALC

11 An awesome rocket ship is in the air and doing cool rocket things. Its velocity $v(t)$ is a differentiable, strictly increasing function. Selected values are given below. Using correct units, explain the meaning of $\int_2^{10} v(t)\,dt$ in the context of this problem. Then, approximate the value of $\int_2^{10} v(t)\,dt$ using the 4 trapezoids indicated by the table.

$\Delta t = 2$

t, sec	2	4	6	8	10
$v(t)$, m/s	12	18	27	38	52



• $\int_2^{10} v(t)\,dt$ is the distance, in meters, the rocket travels over time = 2 to t = 10 seconds.

Area units:

(time, sec)(m/s)

Δx $f(x)$

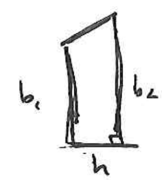
$\cancel{s} \cdot m/\cancel{s}$

= meters

• $\int_2^{10} v(t)\,dt \approx$

$$\approx \left(\frac{12+18}{\cancel{2}}\right)\cancel{2} + \left(\frac{18+27}{\cancel{2}}\right)\cancel{2} + \left(\frac{27+38}{\cancel{2}}\right)\cancel{2} + \left(\frac{38+52}{\cancel{2}}\right)\cancel{2}$$

TRAPEZOIDS



$\left(\frac{b_1+b_2}{2}\right)\cdot h$

$= 12 + 18 + 18 + 27 + 27 + 38 + 38 + 52$

$12 + 36 + 54 + 76 + 52$

$64 + 90 + 76$

$140 + 90$

$\boxed{230 \text{ meters}}$

$64 + 76 = 140$