Key:

I-A7b PRACTICE

1. It's 10am and Frank has already used 8 mb of data on his cell phone. From 10am to midnight (t=24), his data usage rate can be modeled by the differentiable function $f(t) = \sin\left(\frac{\pi}{8}t\right) + 1$ mb/hr. Write an equation that includes an integral that will give the amount of data Frank has used as of midnight. Then, find that amount and include units in your answer.

Net Change Theorem (from the F.T.C.): $F(b) = F(a) + \int_a^b F'(t)\,dt$

future, current, accumulated rate.

$$F(24) = F(10) + \underbrace{\int_{10}^{24} \sin\left(\tfrac{\pi}{8}t\right)+1 \, dt}_{\text{calc.}}$$

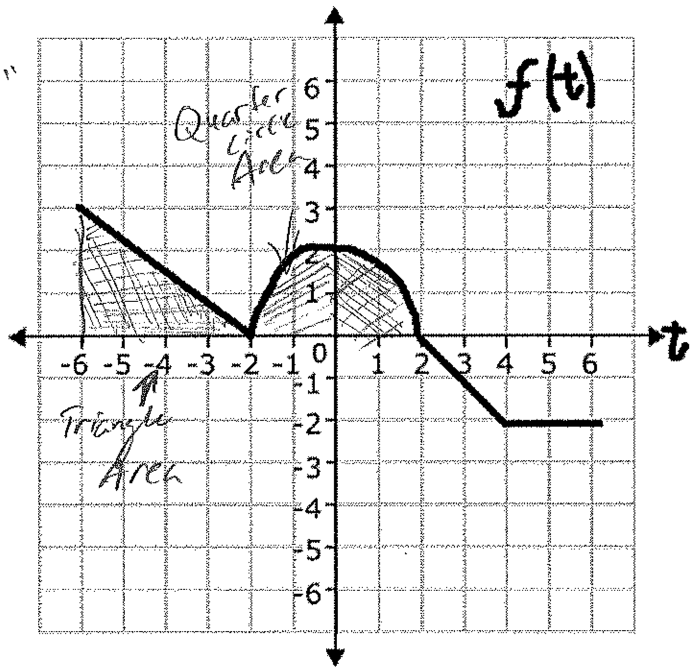
8 mb + 14.746 mb

$\boxed{F(24) = 22.746 \text{ mb}}$

I-U9

The function f(t) is shown over [-6,6] and consists of line segments and a semicircle. "ranking"

Let $G(x) = \int_{-6}^{x} f(t)\,dt$ $\Rightarrow$ $G = \int f$, $G' = f$, $G'' = f'$



2. Find $G(0)$, $G'(0)$, and $G''(0)$.

- $G(0) = \int_{-6}^{0} f(t)\,dt = \underbrace{\tfrac{1}{2}(4)\cdot 3}_{\text{triangle}} + \underbrace{\tfrac{1}{4}\cdot \pi(2)^2}_{\text{Quarter Circle}}$ $\boxed{6+\pi}$
- $G'(0) \Rightarrow f(0) = \boxed{2}$
- $G''(0) = f'(0) = \boxed{0}$ (slope @ $x=0$)

3. Find G(x) relative maxima, if any, over [-6,6]. Justify your answer.

Rel max: where $G'(x) = 0$ & changes from pos → neg.

$G'(x) = f(x) = 0$ this is true at $x = -2$ and $x = 2$

$f(t)$: + | + | − ; t: −2, 2 max

Rel. max @ $x = 2$ because $g'(x) = 0$ and pos → neg sign change. Value here is $\boxed{6 + 2\pi}$

4. Find any points of inflection of G(x). Justify.

$G''(x) = f'(x)$ ← slope of f

$f'(x) = 0$ or undef. @ $x = -2, 0, 2, 4$ terrace points

$g'' = f'$: − | + | − | − | 0 ; t: −2 (I.P.), 0 (I.P.), 2, 4

I.P. @ $x = -2$ and $x = 0$ b/c $g'' = 0$ and changes signs.

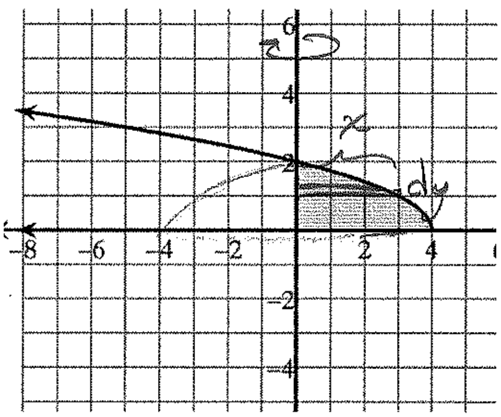
I-U6

5. The FTC states: If $f(x) = \int_a^x g(t)\,dt$, then $f'(x) = \frac{d}{dx}f(x) = \frac{d}{dx}\int_a^x g(t)dt = g(x)$. Explain this in your own words.

If f is the accumulation of the area under $g(t)$ from some fixed point "a" up to a variable value of X, then f', or the rate of change of f, is just g itself.
Differentiation is the inverse of integration.

I-A5a

6. Consider the region bound by y=$\sqrt{4-x}$ and the x and y axes. Set up an integral and then find the volume of the solid generated by revolving this region about the y-axis.



Vol of 1 disk: $\pi(x)^2 dy$

$y=\sqrt{4-x}$
$y^2 = 4-x$
$x = 4-y^2$

$$V = \pi\int_0^2 x^2 \cdot dy$$

$$V = \pi\int_0^2 (4-y^2)^2\, dy$$

calc.

$$= \pi \cdot \frac{256}{15} \approx \boxed{53.617}$$

I-A5b

7. Consider the region between $y = x^2 - 2$ and y=$\sqrt{x} - 2$. Find the volume of the solid generated by revolving this region around the line y=2.

$$V = \pi\int_0^1 (\text{outer radius})^2 - (\text{inner radius})^2\, dx$$

$$V = \pi\int_0^1 (2-(x^2-2))^2 - (2-(\sqrt{x}-2))^2\, dx$$

top − bottom   top − bottom

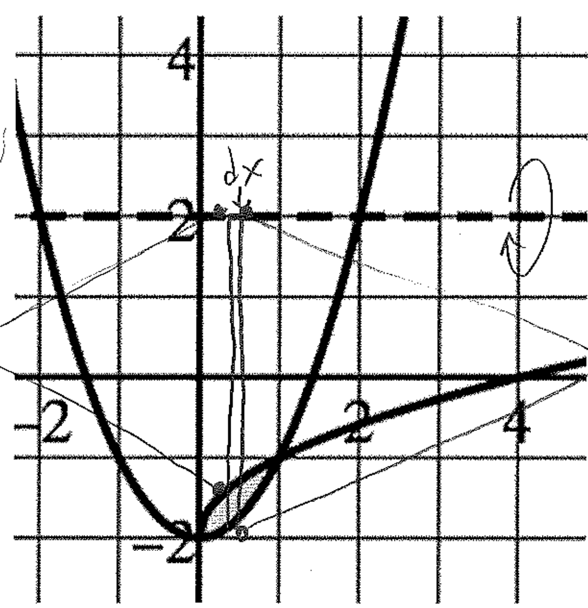
$$V = \pi\int_0^1 (4-x^2)^2 - (4-\sqrt{x})^2\, dx$$

Calc.

$$\pi \cdot \frac{71}{30} \approx 7.435$$

Inner Radius

$2-(\sqrt{x}-2)$



outer radius: $2-(x^2-2)$