Good afternoon, it's nice to be back :)

I will grade your tests as soon as I can and return them Monday

Monday's assessment will have:

- I-U9: FTC Graphically (new)
- I-A4a area under curve (could be dy)
- I-A4b area between curves
- I-U4: FTC algebraically
- I-U7: Properties of Definite Integrals

Tuesday will be a reassessment/upgrade day in class. I will be out Thursday for NYC

Visibly Random Grouping

Thoughts on the test?

- area under a curve
- area between curves
- FTC algebraically
- properties of definite integrals
- FTC 2: Evaluating Definite Integrals
- Finding C
- MRAM
- Riemann Tables

Review

$$\text{Let } f(x) = \int_{-1}^{2x} \frac{1}{4}\, t^3 \, dt \qquad \text{Find } f'(4) \text{ and } f''(4)$$

$$f'(x) = \frac{1}{4}(2x)^3 \cdot 2$$

$$\cancel{\tfrac{1}{4}} \cdot \overset{2}{\cancel{8}}x^3 \cdot 2$$

$$f'(x) \quad 4x^3$$

$$f'(4) = 4\left(\cancel{4^3}^{\,64}\right) = 256$$

	10	2
10	100	20
6	60	12

$$f''(x) = 12x^2$$

$$f''(4) = 12(16) = \underline{192}$$

Net Change Theorem

FTC II:

$$\int_a^b f'(x)\,dx = f(b) - \cancel{f(a)}$$

$$+f(a) \qquad\qquad \cancel{+f(a)}$$

$$f(a) + \int_a^b f'(x)\,dx = f(b)$$

$$f(b) = f(a) + \int_a^b f'(x)\,dx$$

future value — initial value — accumulated changes from $a \to b$

$=$

$+ \int$

Future value = Initial value + accumulated change

In terms of position/velocity:

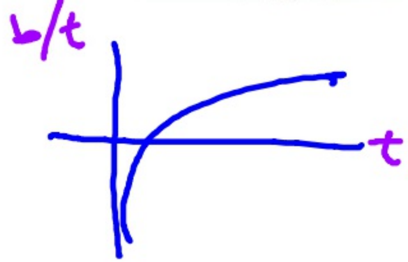
$$x(T) = x(a) + \int_a^T v(t)\,dt$$

A honey bee population starts with 30 bees and grows at a rate of B'(t)=ln(12t+1) bees per day. How many bees are there after 1 day?

After 3 days?

$$B(T) = B(0) + \int_0^T B'(t)\,dt$$

$$B(1) = 30 + \underline{\int_0^1 \ln(12t+1)\,dt}$$



$$30 + 1.779$$

$$\boxed{B(1) \approx 31}$$

$$B(3) = 30 + \int_0^3 \ln(12t+1)\,dt \approx \boxed{38 \text{ bees}}$$

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

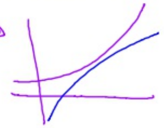
$M \rightarrow R \rightarrow R'$

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.



a) $R(6) = 5\sqrt{6}\cdot\cos\left(\frac{6}{5}\right) = 4.438 > 0$

$R(6) > 0$, so mos. pop is increasing.

b) $R'(6) = -1.913$. Mosq. are dec. increasing b/c

$R'(6) < 0$, so Mosq. pop. is concave down.

c) $M(31) = 1000 + \int_0^{31} R(t)\,dt$

$= 1000 + -35.665 \longrightarrow$ 964 mosq.

d) ABS. MAX: C.N. and endpts.

$M(0) = 1000$

$M(31) = 964$

$M(T) = 1000 + \int_0^{T} R(t)\,dt$

$M'(T) = 0 + R(T) = 0$

$T = 7.854 \leftarrow$ input

$M(7.854) = 1039.35$

1039 mosq.

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

$$A(t) = 30 + \underbrace{8t}_{\text{water in}} - \underbrace{\int_0^t \sqrt{x+1}\,dx}_{\text{water out}}$$

or . . .

a.) $\int_0^3 (t+1)^{1/2}\,dt$

$= \frac{2}{3}(t+1)^{3/2}\Big]_0^3$

$\frac{16}{3} - \frac{2}{3} = 14/3$ gal

b.) $A(3) = 30 + 24 - \frac{14}{3} \rightarrow 54 - \frac{14}{3}$ gal

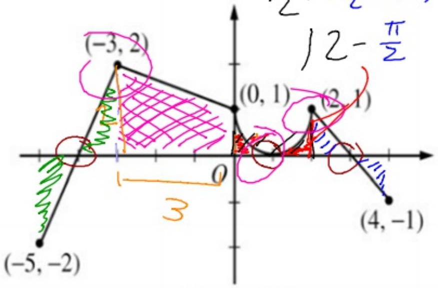
c.) $A(t) = 30 + 8t - \int_0^t \sqrt{x+1}\,dx$

d.) tbd.

FTC Graphically (no calculator)

$2-\frac{1}{2}\pi(1)^2$

$2-\frac{\pi}{2}$



Graph of f

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

$g' = f(x)$

$g'' = f'(x)$

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

a) $g(0) = \int_{-3}^{0} f(t)\,dt = 4.5$

$g'(0) = 1$

b) g has rel. max @ $x = 3$ b/c $g' = 0$ and changes $+ \to -$.

c) Abs Min: Endpts + C.N. ($x=-4$ rel min)

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -\boxed{\int_{-4}^{-3} f(t)} = -1$

$g(-5) = \int_{-3}^{-5} = -\int_{-5}^{-3} f(t) = 0$

$g(4) = \int_{-3}^{4} f(t) = 4.5 + 2 - \frac{\pi}{2}$

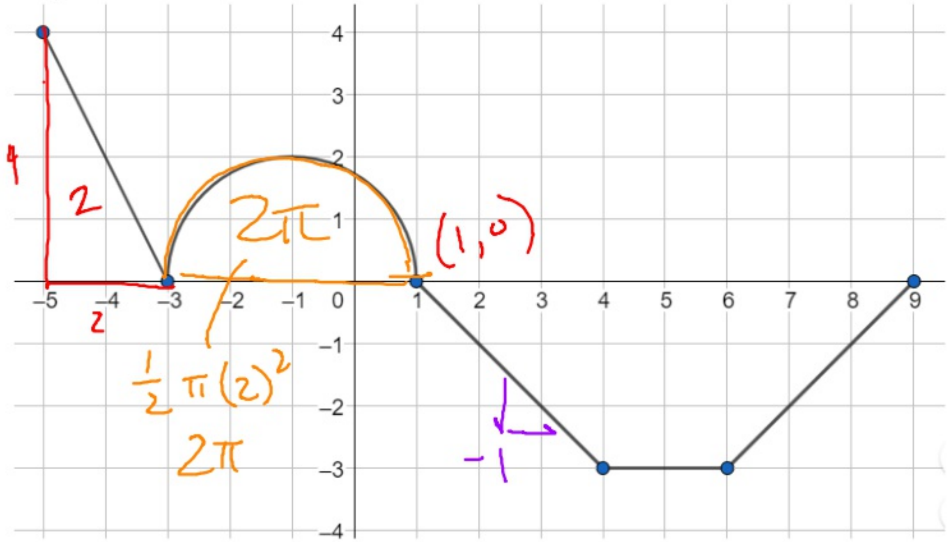
$6.5 - \frac{\pi}{2} > 0$

Abs. Min

g has an abs min @ $x=-4$ based on these calc. and that value is -1.

d) $x = -3, 2, 1$

Bonus Content!



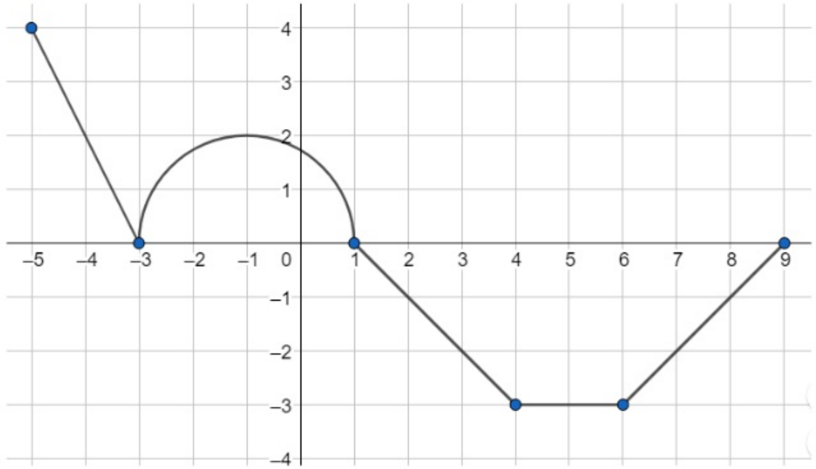
Let $H(x) = \int_{-5}^{x} g(t)\,dt$

$H' = g$

$H'' = g'$

Find H(1), H'(1), and H"(3).

- $H(1) = \int_{-5}^{1} g(t)\,dt = 2 + 2\pi$
- $H'(1) = g(1)$ ← value of g @ $x=1$ → 0.
- $H''(3) = g'(3)$ ← slope of g @ $x=3$ → -1



Let $H(x) = \int_{-5}^{x} g(t)dt$

$H' = g$

$H'' = g'$

Find the x-coordinates of any relative maxima/minima for H. Justify

H' sign change → g sign change

H has rel. max @ $x=-1$ b/c H' $(=g) = 0$, changes $+ \to -$ sign.

H has no rel. min in the interval.

Find the x-coordinates of any of H's inflection points. Justify

H'' sign change → g' sign change.

H has f.p. @ $x = -3, -1$ b/c H'' $(= g')$ changes sign there.

(Note that $x=4$ & $x=7$ are not $+ \to -$ sign changes).