Good afternoon: no warm up, check hw answers now
we'll randomize and look at area some more, then accumulation functions

Answers to Area Between Curves

1) $= \frac{16}{3} \approx 5.333$

2) $= \frac{253}{12} \approx 21.083$

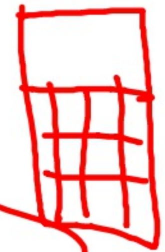
3) $= \frac{38}{3} \approx 12.667$

4) $= 4$

5) $= 16$

6) $= 9$

will need Calc.

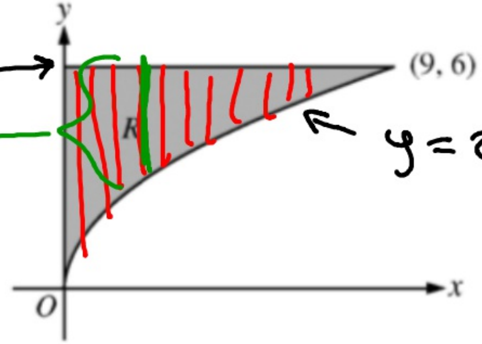


Next test: Tuesday
last Q3 test: March 11

visibly random grouping

Some more work with Area

no calc



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

$$\int_0^9 6 - 2\sqrt{x}\, dx$$

$-2x^{1/2}$

$$6x - \frac{4}{3}x^{3/2}\Big|_0^9$$

$9^{3/2}$

$(9^{1/2})^3$

$$54 - \frac{4}{3}\cancel{(9^{3/2})}$$

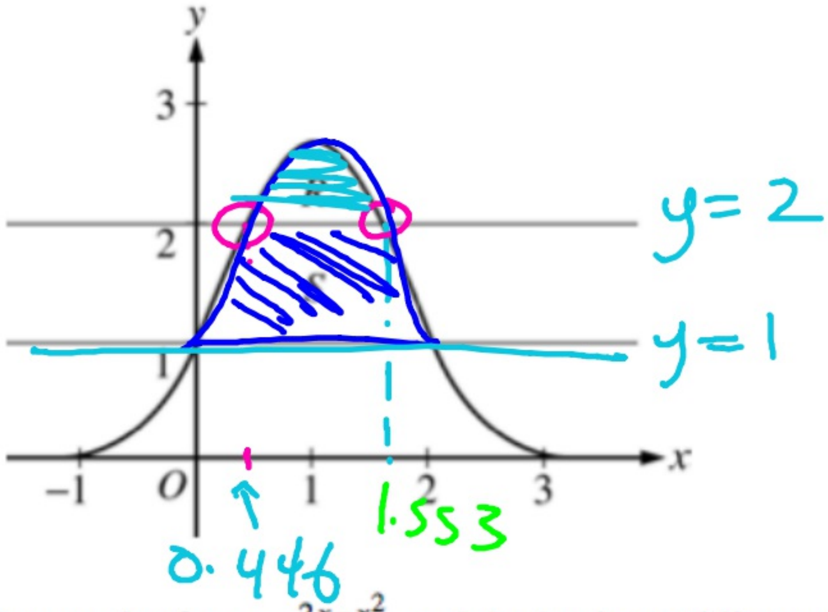
27

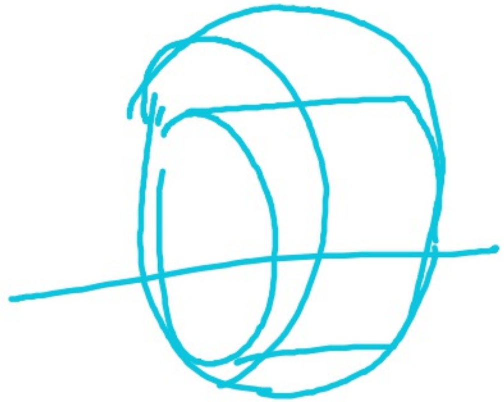
$$54 - 36 = 18$$

(a) Area $= \int_0^9 (6 - 2\sqrt{x})\, dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big|_{x=0}^{x=9} = 18$

3 : { 1 : integrand
1 : antiderivative
1 : answer

Calc OK





$$y = e^{2x-x^2}$$

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

$$\int_{0.446}^{1.553} \left[e^{2x-x^2} - 2 \right] dx = 0.514$$

$$S = \underbrace{\int_0^2 \left[e^{2x-x^2} - 1 \right] dx}_{(R+S)} - \underbrace{0.514}_{(R)}$$

$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$
Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) $e^{2x-x^2} = 1$ when $x = 0, 2$

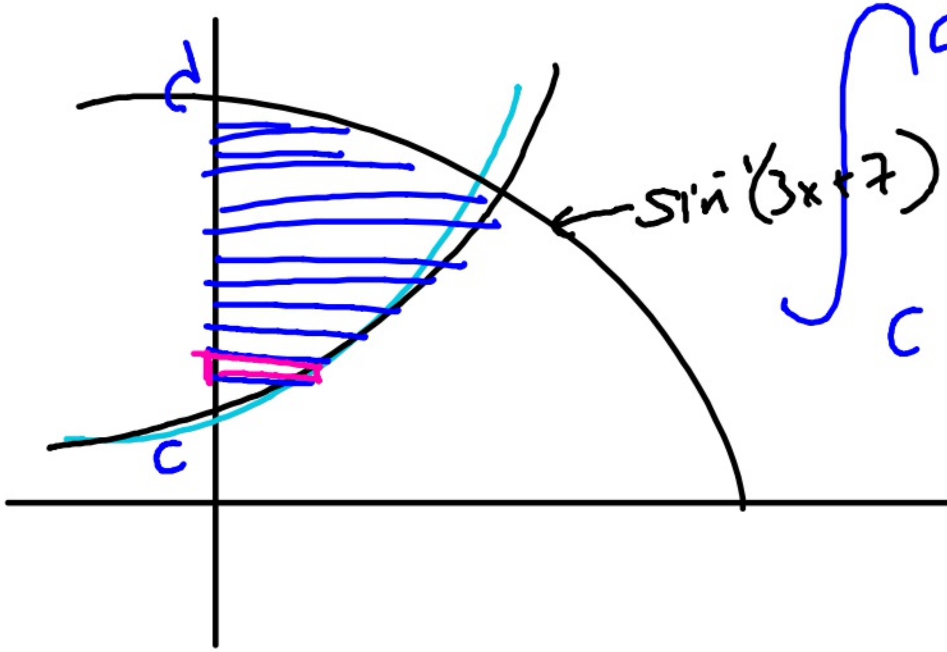
Area of $S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx -$ Area of R
$= 2.06016 -$ Area of $R = 1.546$

OR

$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q - P) \cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$
$= 0.219064 + 1.107886 + 0.219064 = 1.546$

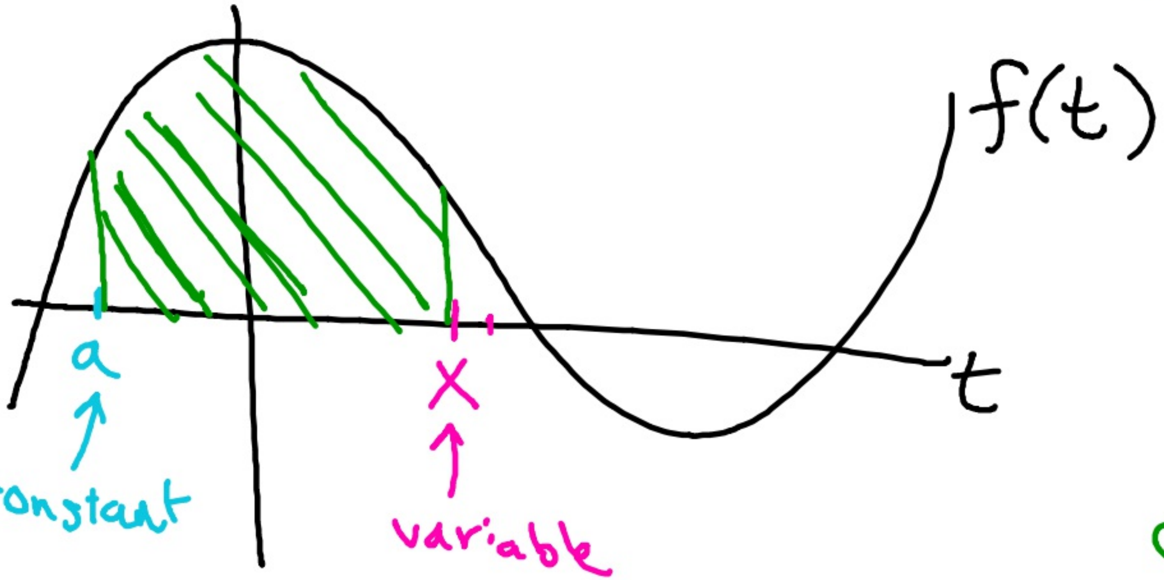
$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

Tomorrow's mini lesson: dy Area



$$\int_c^d \qquad dy$$

Accumulation Functions....a new category of functions



$$F(x) = \int_a^x f(t)\,dt$$

accumulation funct.

as x varies, the area under f changes
thus the value of F changes

https://www.desmos.com/calculator/lvu8rfhs1k

Function Types You've Studied

1. constant/linear
2. absolute value
3. quadratic
4. cubic
5. quartic
6. polynomial
7. rational
8. exponential
9. logarithmic
10. trigonometric
11. inverse trigonometric
12. accum. functions.

Also known as "functions defined by integrals"

They ask the question,
how much space is under this *other* function?

(this has applications well beyond area: think velocity)

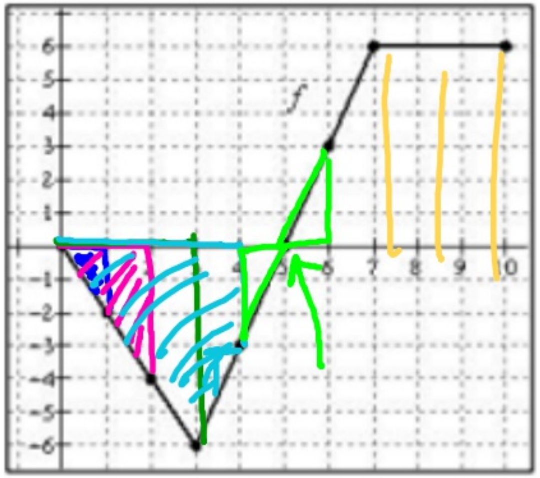
[redacted]

Better Understanding Accumulation Functions

Let $F(x)=\int_0^x f(t)\,dt$, where the graph of f is shown at right. Complete the following chart.

$$F(x)=\int_0^x f(t)\,dt$$

$$F(2)=\int_0^2 f(t)\,dt$$



$\leftarrow f$

$$\int_0^6=\int_0^5+\int_5^6$$

$-15+1.5$

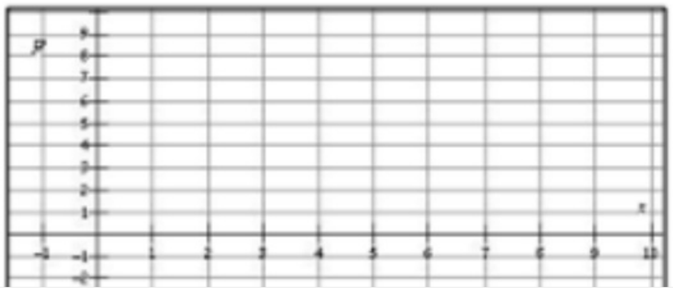
x	0	1	2	3	4	5	6	7	8	9	10
$F(x)$	0	-1	-4	-9	-13.5	-15	-13.5	-9	-3	3	9

On what interval is f positive?

On what interval is f negative?

On what interval is F increasing?

On what interval is F decreasing?



x	0	1	2	3	4	5	6	7	8	9	10
$F(x)$	0	-2	-4	-9	-13.5	-15	-13.5	-9	-3	3	9

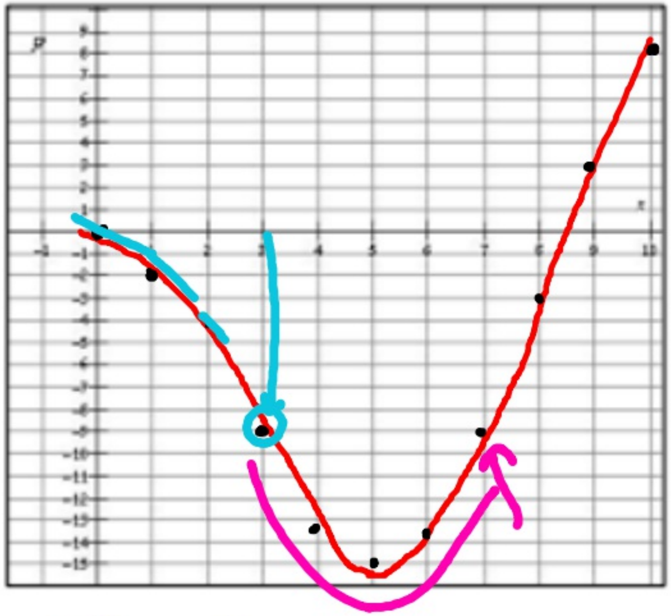
On what interval is f positive?

On what interval is f negative?

On what interval is F increasing?

On what interval is F decreasing?

Plot the points $(x, F(x))$ on the grid provided.



On what interval is f decreasing? $(0,3)$

On what interval is f increasing? $(3,7)$

On what interval is the graph of F concave down? $(0,3)$

On what interval is the graph of F concave up? $(3,7)$

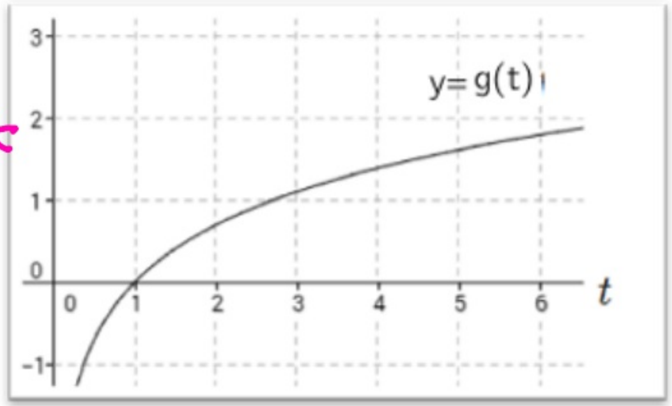
What is the relationship between f and F?

$F' = f$

We are now ready to state/prove
the Fundamental Theorem of Calculus

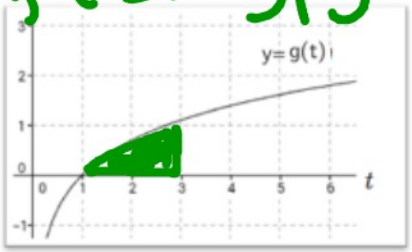
Let $f(x) = \int_a^x g(t)dt$ for the graph of g(t) shown.

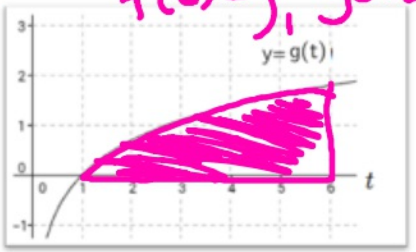
Let a=1 for simplicity. Sketch in $f(3)$ and $f(6)$



$f(3) = \int_1^3 g(t)dt$

$f(6) = \int_1^6 g(t)dt$





Let us investigate an "increment" of the area. Namely, the part added on between when you move from a value of x a value of $x + \Delta x$

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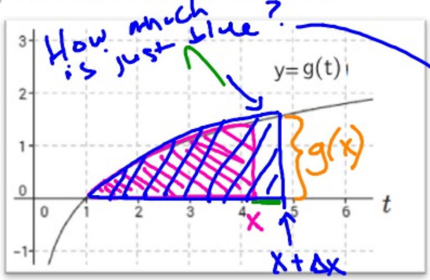

Find use geometry to find the area of the incremental "rectangle"

(1) $\underbrace{g(x)}_{\text{height}}\ \underbrace{\Delta x}_{\text{(base)}}$

Find the area using the accumulation function.

(2) $f(x+\Delta x) - f(x)$

Since they both describe the same space, items (1) and (2) should be equal. So:

$$f(x+\Delta x) - f(x) = g(x)\,\Delta x$$

divide by Δx

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = g(x)$$

take $\lim_{\Delta x \to 0}$

$$\boxed{\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}} = \lim_{\Delta x \to 0} g(x)$$

limit def. of deriv.

$$f'(x) = g(x)$$

stopped here Tues PM

$$\frac{d}{dx} f(x)$$

$$\frac{d}{dx}\int_a^x g(t)\,dt = g(x).$$

$$\frac{d}{dx}\int_a^x g(t)\,dt = g(x)$$

derivative of
an accumulation
function is the function
being accumulated

FTC part 1

If $g(x)$ is continuous on $[a, b]$ and $f(x) = \int_a^x g(t)dt$

Then $f'(x) = g(x)$

HW
#7-12 on back of area handout
number answers at mcalc.weebly.com

test Tuesday