

Good afternoon: warm up in notebooks

Find $F'(x)$ for each

$$F(x) = \int_e^x [t^3 - 3t^2 + 4t - \ln(t)] dt$$

$$\rightarrow F'(x) = x^3 - 3x^2 + 4x - \ln(x)$$

$$F(x) = \int_x^{\infty} \operatorname{arcsec}(3t) dt = \emptyset$$

$$F(x) = \int_x^{-2} \frac{2t}{3t-4} dt$$

$$\left. -\frac{1}{3x} F(x) - \frac{1}{3x} \right|_{-2}^x + \int_{-2}^x \frac{2t}{3t-4} dt$$

$$F'(x) = -\frac{2x}{3x-4}$$

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Reminders:

- tutoring TODAY 4-5p
- Q3 ends 3/16!

FTC with the Chain Rule

$$F(x) = \int_a^{h(x)} g(t) dt$$

$$\frac{dF}{dx} = \frac{d}{dx} \int_a^{h(x)} g(t) dt$$

$$\frac{dF}{dx} = \frac{d}{dh} \int_a^{h(x)} g(t) dt * \frac{dh}{dx}$$

$$f'(h(x)) \cdot h'(x)$$

$$\frac{dF}{dx} = \frac{dF}{dh} * \frac{dh}{dx}$$

chain
rule

$$\left[\frac{dF}{dx} = g(h(x)) * \frac{dh}{dx} \right]$$

Example:

$$F(x) = \int_7^{x^3} \sec\left(\frac{5t}{3\pi}\right) dt$$

F'(x)?

$$\sec\left(\frac{5 \cdot x^3}{3\pi}\right) \cdot 3x^2$$

$$F(x) = \int_{-2x}^{x^4} 12t^2 - 3 \, dt$$

$$F(x) = \int_{-2x}^5 12t^2 - 3 \, dt + \int_5^{x^4} 12t^2 - 3 \, dt$$

$$f(x) = - \int_5^{2x} 12t^2 - 3 \, dt + \int_5^{x^4} 12t^2 - 3 \, dt$$

$$\begin{aligned} F' &= - \left[[12(-2x)^2 - 3] \cdot -2 \right] + \left[12(x^4)^2 - 3 \right] [4x^3] \\ &\quad - \left[(48x^2 - 3) \cdot -2 \right] + \left[12x^8 - 3 \right] [4x^3] \\ &\quad - \left[-96x^2 + 6 \right] + \overbrace{48x^1 - 12x^3}^{96x^2 - 6} \end{aligned}$$

HW sols

$$g(10) = 0$$

$$g(-2) = -8$$

$$g(8) = 4$$

$$g(-4) = -4$$

$$73. g(x) = \int_0^x f(t) dt$$

$$(a) g(0) = \int_0^0 f(t) dt = 0$$

$$g(2) = \int_0^2 f(t) dt \approx 4 + 2 + 1 = 7$$

$$g(4) = \int_0^4 f(t) dt \approx 7 + 2 = 9$$

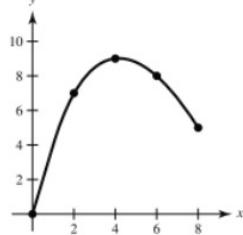
$$g(6) = \int_0^6 f(t) dt \approx 9 + (-1) = 8$$

$$g(8) = \int_0^8 f(t) dt \approx 8 - 3 = 5$$

(b) g increasing on $(0, 4)$ and decreasing on $(4, 8)$

(c) g is a maximum of 9 at $x = 4$.

(d)



$$74. g(x) = \int_0^x f(t) dt$$

$$(a) g(0) = \int_0^0 f(t) dt = 0$$

$$g(2) = \int_0^2 f(t) dt = -\frac{1}{2}(2)(4) = -4$$

$$g(4) = \int_0^4 f(t) dt = -\frac{1}{2}(4)(4) = -8$$

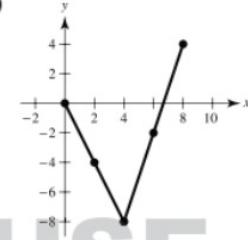
$$g(6) = \int_0^6 f(t) dt = -8 + 2 + 4 = -2$$

$$g(8) = \int_0^8 f(t) dt = -2 + 6 = 4$$

(b) g decreasing on $(0, 4)$ and increasing on $(4, 8)$

(c) g is a minimum of -8 at $x = 4$.

(d)



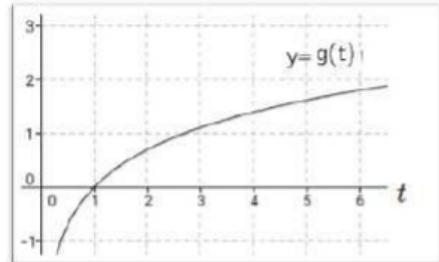
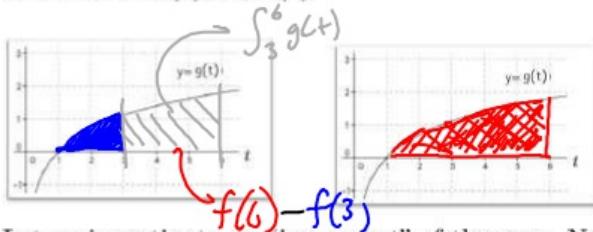
Proving the Fundamental Theorem of Calculus

A Visual and Algebraic Proof of The Fundamental Theorem of Calculus

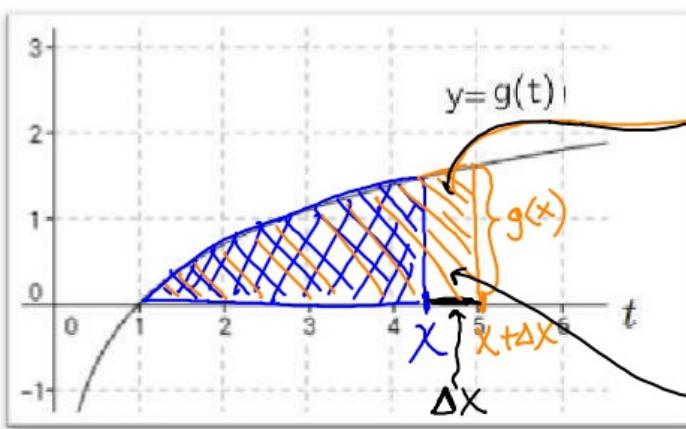
$$a=1$$

Let $f(x) = \int_a^x g(t) dt$ for the graph of $g(t)$ shown.

So sketch in $f(3)$ and $f(6)$



Let us investigate an "increment" of the area. Namely, the part added on between when you move from a value of x a value of $x + \Delta x$



Find use geometry to find the area of the incremental "rectangle"

$$(1) \quad g(x) \Delta x$$

height · base

Find the area using the accumulation function.

$$(2) \quad f(x+\Delta x) - f(x)$$

orange blue

blue

Since they both describe the same space, items (1) and (2) should be equal. So:

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{g(x) \cdot \Delta x}{\Delta x}$$

) divide Δx

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} g(x)$$

) shrink Δx

limit def. of deriv. $f'(x) = g(x)$

$$\frac{d}{dx} [f(x)] = g(x)$$

$$\frac{d}{dx} \int_a^x g(t) dt = g(x)$$

derivative of an accumulation function
is the function itself.

By the result on the bottom of pg1: derivative is the inverse of definite integration.

By common sense (construction) the derivative is the inverse of the antiderivative.

THEREFORE:

FTC Part 2

Now, why is this true: $\int_a^b g(x)dx = f(b) - f(a)$ where $f'(x) = g(x)$

Let $f(x) = \int_a^x g(t) dt$

$$(I) f(a) = \int_a^a g(t) dt = 0$$

$$(II) f(b) = \int_a^b g(t) dt$$

by Def. Int. Properties.

$$\int_a^b g(t) dt = \int_a^a g(t) dt + \int_a^b g(t) dt$$

$$\int_a^b g(t) dt = - \int_a^a g(t) dt + \int_a^b g(t) dt$$

$$\int_a^b g(t) dt = - \underbrace{f(a)}_{\text{by (I)}} + \underbrace{f(b)}_{\text{by (II)}}$$

$$\int_a^b g(t) dt = f(b) - f(a)$$

Q.E.D.

Homework:

p. 290 #82-92 (even) , 103, 104

p. AP4-1 (comes after p316) #2 and #4