Good afternoon: no warm up, check hw
we will randomize then continue where we left off Tues

7) $= \frac{97}{6} \approx 16.167$

8) $= \frac{59}{6} \approx 9.833$

9) $= 9$

10) $= 24$

11) $= \frac{253}{24} \approx 10.542$

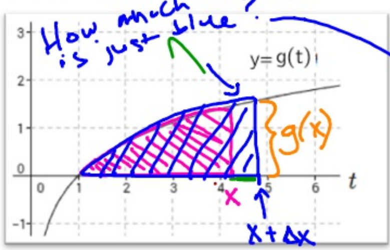
12) $= \frac{16}{3} \approx 5.333$

1 Test: Tuesday

2 next test: 3/11 (MON.)
(A · DAY)

3 Last day to retake: 3/13. W

Let us investigate an "increment" of the area. Namely, the part added on between when you move from a value of x a value of $x + \Delta x$



Find use geometry to find the area of the incremental "rectangle"

(1) $\underbrace{g(x)}_{\text{height}}\ \underbrace{\Delta x}_{\text{(base)}}$

Find the area using the accumulation function.

(2) $f(x+\Delta x) - f(x)$

Since they both describe the same space, items (1) and (2) should be equal. So:

$$f(x+\Delta x) - f(x) = g(x)\,\Delta x$$

divide by Δx

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = g(x)$$

take $\lim_{\Delta x \to 0}$

$$\boxed{\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}} = \lim_{\Delta x \to 0} g(x)$$

limit def. of deriv.

$$f'(x) = g(x)$$

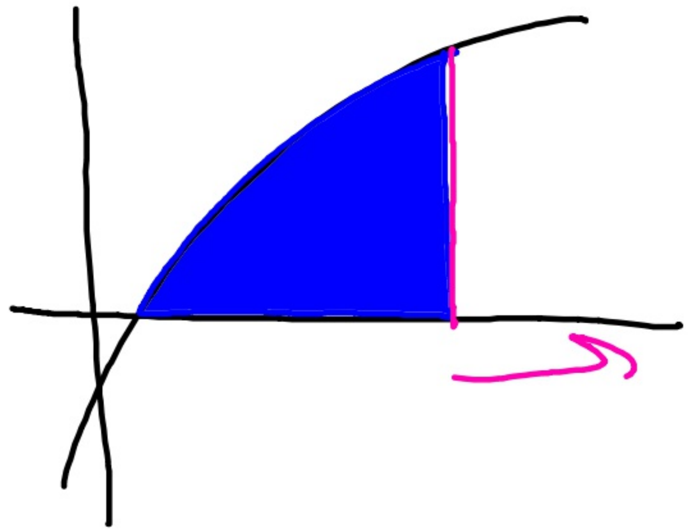
stopped here Tues PM

$$\frac{d}{dx} f(x)$$

$$\frac{d}{dx}\int_a^x g(t)\,dt = g(x).$$

$$\underbrace{\frac{d}{dx}\int_a^x g(t)\,dt}_{} = g(x)$$

derivative of
an accumulation
function is the function
being accumulated



FTC part 1

If $g(x)$ is continuous on $[a, b]$ and $f(x) = \int_a^x g(t)dt$

Then $f'(x) = g(x)$

- Deriv is inverse of Def. Int. (Area)
- Deriv. is inverse of antiderivative.

Def Int (Area) $\equiv$ antiderivative

FTC Part 2

Now, why is this true?? $\int_a^b g(x)dx = f(b) - f(a)$ where $g(x) = f'(x)$

Solipsism

Let $f(\cancel{x}) = \int_a^{\cancel{x}} g(t)\,dt$

I. observe that f(a) = $\int_a^a g(t)\,dt$ $(= 0)$

II. observe that f(b) = $\int_a^b g(t)\,dt$

$$\int_a^b g(t)dt = \int_a^a g(t)dt + \int_a^b g(t)dt$$

$$\int_e^f = \int_e^g + \int_g^f$$

$$\int_a^b = -\int_b^a$$

$$\underbrace{-\int_a^a g(t)dt}_{\text{by I}} + \underbrace{\int_a^b g(t)dt}_{\text{by II}}$$

$$-f(a) + f(b)$$

$$\int_a^b g(t)dt = f(b) - f(a)$$

Q.E.D.

How to "use" the FTC 1

$$\left\{ \frac{d}{dx}\int_a^x \boxed{g(t)}\,dt = \boxed{g(x)} \quad \text{Used to take deriv. of accum. functions.} \right\}$$

ex/ Let $\frac{d}{dx}F(x) = \cancel{\frac{d}{dx}}\int_3^x \sin^2(e^{3t}+2t)\,dt$

$F'(x)$?

$$F'(x) = \sin^2(e^{3x}+2x)$$

$$G(x) = \int_x^5 \left[\ln|2t + 4t^2|\right] dt$$

$$G(x) = -\int_5^x$$

$$G'(x) = -\ln|2x + 4x^2|$$

Chain Rule

Let $H(x) = \int_{-4}^{2x^2} \cos^{-1}(4t^2+1)\,dt$

$H'(x)$?

$$H'(x) = \cos^{-1}\left(4(2x^2)^2+1\right) \cdot 4x$$

$$H'(x) = 4x \cdot \cos^{-1}\left(16x^4+1\right)$$

$$Q(x) = -\int_{2x^3}^{5} e^{-2t^2}\,dt$$

$$Q'(x) = ? \quad -\,e^{-2(2x^3)^2} \cdot 6x^2$$

$$\boxed{-6x^2 e^{-8x^6}}$$

Two variable bounds??

$$F(x) = \int_{x+2}^{x^2} \sin(t)\,dt$$

$\boxed{F'(x)\ ?}$

$$= \int_{x+2}^{420} \sin(t)\,dt + \int_{420}^{x^2} \sin(t)\,dt$$

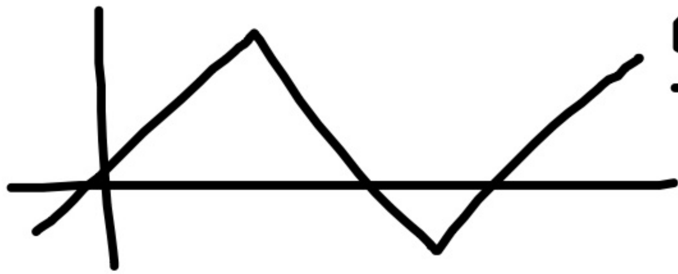
$$F(x) = -\int_{420}^{x+2} \sin(t)\,dt + \text{''} \quad \text{''}$$

$$F'(x) = -\sin(x+2)\cdot 1 + \sin(x^2)\cdot 2x$$

Big picture implications of the FTC

$$f(x) = \int_a^x g(t)\,dt$$

area under g



FTC:

$$f'(x) = g(x)$$

value of g

$$f''(x) = g'(x)$$

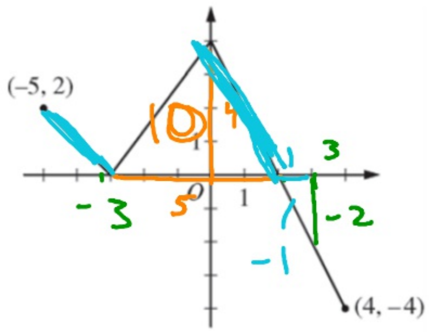
Slope of g

$\vdots \qquad \vdots$

a) $\int_{-3}^{3} f(t)\,dt$

$= \overset{10}{} + \overset{-1}{}$

$= 9$



Graph of f

$g(x) = \int_{-3}^{x} f(t)\,dt$

$g'(x) = f(x)$ value

$g''(x) = f'(x)$ slope

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$. 9

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

$f\ \cancel{g'}\ +$ $\quad f'\ \cancel{g''}\ -$

(c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

b) g is Inc./C.D over

(−5, −3) and (0, 2) b/c

g′ is positive and g″ negative

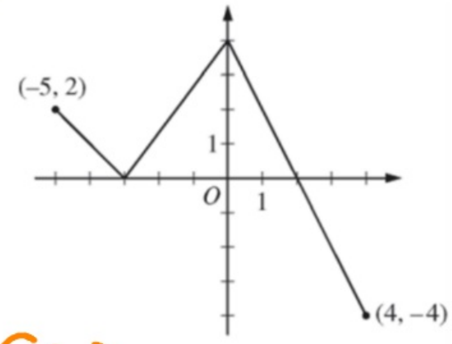
(c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.

$$\frac{f}{g}$$

$$\frac{f'g - fg'}{g^2}$$

$$h'(x) = \frac{g'(x)\cdot 5x - g(x)\cdot 5}{25x^2}$$

$$h'(3) = \frac{\overset{-2}{\cancel{g'(3)}}\cdot 15 - \overset{9}{g(3)}\cdot 5}{25\cdot 9} = \frac{-30 - 45}{25\cdot 9}$$



Graph of f

$g'(3) = f(3)$

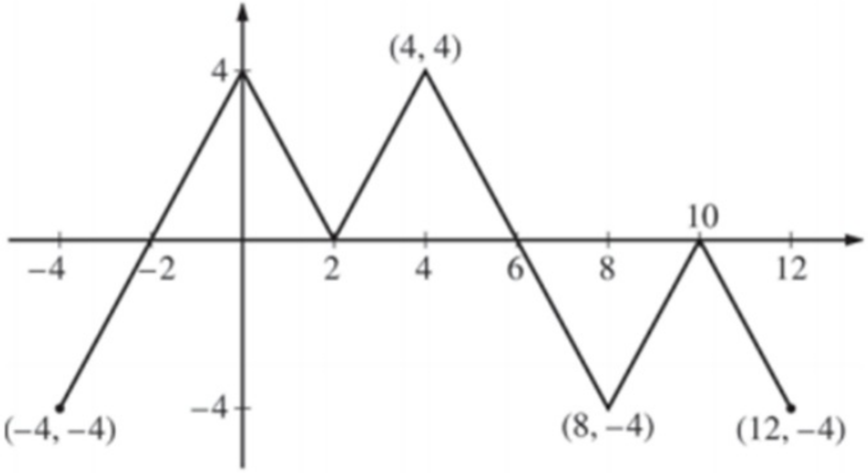
$= -2$

(d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

$g = \int_2^x f$

$g' = f$

$g'' = f'$



Graph of f

. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

sign change in $\cancel{g'}$ f ?

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

Neither. $f = g'$ doesn't change signs.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

sign change in $\cancel{g''}$ f' ?

Yes, sign change in $f'(g'')$.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

What's on Tuesday's test

4 New Skills

I-A4a: area under a curve (no calc)

I-A4b: area between curves (calc ok)

I-U4: Using the FTC1 (no calc)

I-U7: Properties of Definite Integrals (no calc)

4 Old Skills

I-U3b: Midpoint/Trapezoid approx, function (calc ok)

I-U3c: Approximating integral with table (no calc)

I-U5: Evaluate definite integral using FTC2 (no calc)

I-A3: Finding C (possibly a motion problem) (no calc)

Additional practice for these skills:

I-A4a: p 288 #35-38

I-A4b: p 442 #1-10

I-U4: p 290 #81-92

I-U7: p 313 #41, 42

I-U3b:
- mid: p 264 #61-64
- trap p 310 #1-8

I-U3c: p 311 #39

I-U5: p 313 #43-48

I-A3: p 312 #9-12