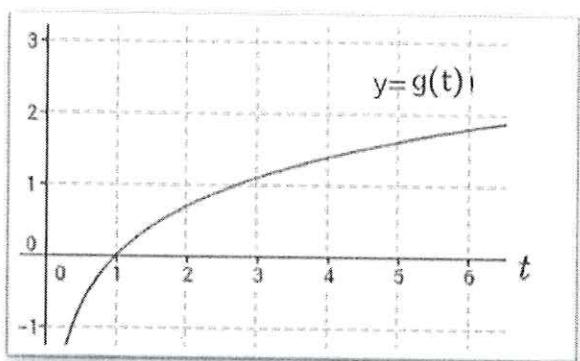
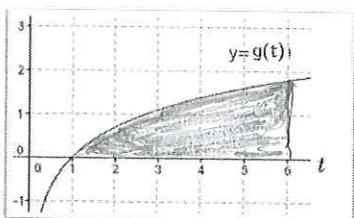
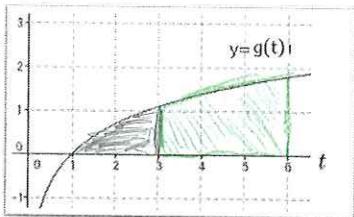


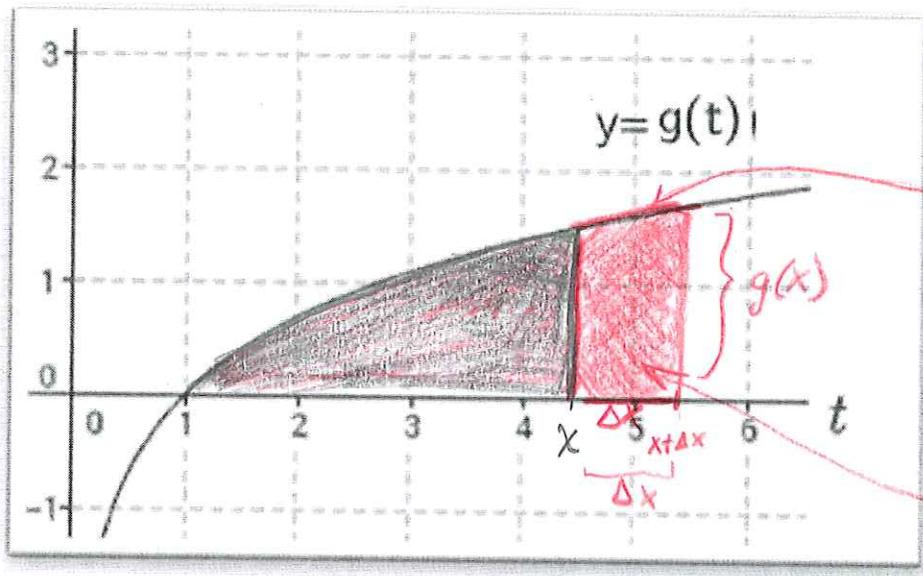
# The Fundamental Theorem of Calculus

Let  $f(x) = \int_a^x g(t)dt$  for the graph of  $g(t)$  shown.  
So sketch in  $f(3)$  and  $f(6)$



Note: to get the green space:  
Area  $= f(6) - f(3)$   
[or,  $\int_3^6 g(t)dt$ ]

Let us investigate an "increment" of the area. Namely, the part added on between when you move from a value of  $x$  a value of  $x + \Delta x$



Find use geometry to find the area of the incremental "rectangle"

(1)  $g(x) \cdot \Delta x$   
height Base

Find the area using the accumulation function.

(2)  $\underbrace{f(x+\Delta x) - f(x)}_{\text{grey-red and red space}} = \text{grey space}$

Since they both describe the same space, items (1) and (2) should be equal. So:

$$f(x+\Delta x) - f(x) = g(x) \Delta x$$

Divide by  $\Delta x$

$$\lim_{\Delta x \rightarrow 0} \left( \frac{f(x+\Delta x) - f(x)}{\Delta x} \right) = g(x) \quad \text{D.L.}$$

Take limit as  $\Delta x \rightarrow 0$   
to improve accuracy;

FTC

$$\frac{d}{dx} \left[ \int_a^x g(t)dt \right] = g(x)$$

Substitute basal  
on line 1  
of paper

$$\lim_{\Delta x \rightarrow 0} \left( \frac{f(x+\Delta x) - f(x)}{\Delta x} \right) = g(x)$$

$$f'(x) = g(x)$$

$$\frac{d}{dx} [F(x)] = g(x)$$

(Definite)  
Derivative and Integration  
are inverses

(1) By Riemann definition  $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_a^b f(x) dx$ . So, integration computes area.

(2) By the result on the bottom of the last page: derivative is the inverse of integration. <sup>Definite</sup>

(3) By common sense (construction) the derivative is the inverse of the antiderivative.

THEREFORE:

Antiderivative is the same  
as definite integral, so  
it computes area.

## FTC Part 2

Now, why is this true:  $\int_a^b g(x) dx = f(b) - f(a)$

Let  $f(x) = \int_a^x g(t) dt$

• Find  $f(a)$

$$(i) f(a) = \int_a^a g(t) dt = 0$$

• Find  $f(b)$

$$(ii) f(b) = \int_a^b g(t) dt$$

• What is

$\int_a^b g(t) dt$  ? Well, by Properties of Def. Integrals

$$\int_a^b g(t) dt = \int_a^a g(t) dt + \int_a^b g(t) dt$$

Property: flip limits of integration  $\int_a^a g(t) dt + \int_a^b g(t) dt$

$$\underbrace{\int_a^b g(t) dt}_{\text{see Ques (ii)}} - \underbrace{\int_a^a g(t) dt}_{\text{see Ques (ii)}} \quad ) \text{ commutative property; } \quad )$$

$$\int_a^b g(t) dt = f(b) - f(a)$$

"Symbolically" Let  $\cancel{\ll}$  mean "inverse, or opposite"

(3) Derivative  $\cancel{\ll}$  Antiderivative

(2) Derivative  $\cancel{\ll}$  Def. Integral

therefore Antiderivative must  
be the same as  
Def. Integration.

Example

$$\frac{d}{dx} \int_5^x \cos^2(t) - t^3 + \sqrt[5]{t} dt \\ = \cos^2(x) - x^3 + \sqrt[5]{x}$$