Good Afternoon

Happy π day!

We will assess soon after the bell rings. Be prepared with any questions you have before then.

Reminders:

- Last day to re-assess for Q3 is Friday

$$\frac{d}{dx}F = \frac{d}{dx}\int_{x}^{x^2} e^{2t}\,dt$$

F'?

$$F(x) := \int_a^x f(t)\,dt$$

$$F'(x) = f(x)$$

FTC

$$-\int_{x}^{0} e^{2t}\,dt + \int_0^{x^2} e^{2t}\,dt$$

$$-\int_0^{x} e^{2t}\,dt + \int_0^{x^2} e^{2t}\,dt$$

then take the derivative

$$F' = -e^{2x} + e^{2x^2}\cdot 2x$$

$$\frac{d}{dx}\left[g(x)\right] = \frac{d}{dx}\int_{-3}^{4x} \sin^2(5t)\,dt$$

$$g'(x)\ ? = \sin^2(5\cdot 4x)\cdot 4$$

Chain Rule

$$\boxed{4\sin^2(20x)}$$

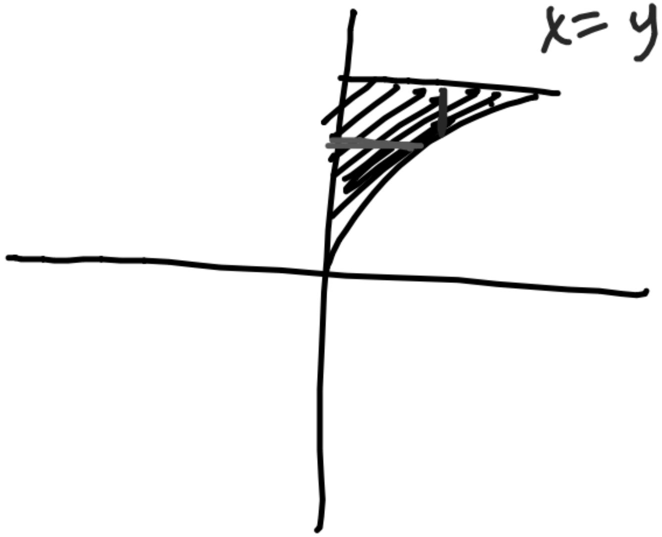
Given: $\int_3^5 f(x)\,dx = 2$

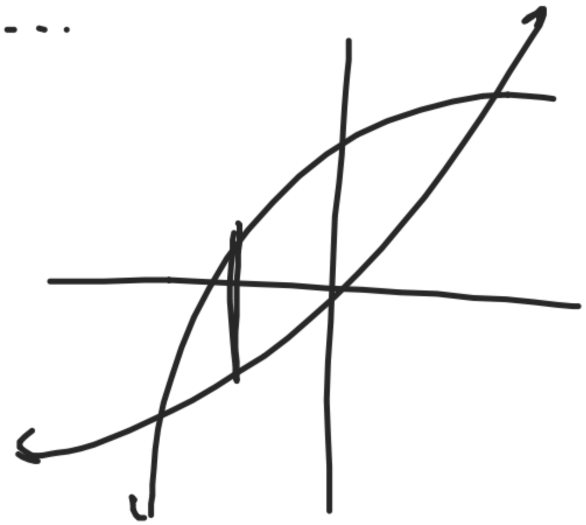
$$\int_a^b c\cdot f(x) = c\int_a^b f(x)$$

Find: $\int_5^3 7f(x)\,dx = ? = -\int_3^5 7f(x)\,dx$

$$-7\boxed{\int_3^5 f(x)} = -14$$

$x = y^2$ . . . . .





Questions?

- Applying the FTC
- Definite Integral Properties
- Average Value
- Area Between Curves

Since several skills are being assessed only once in Q3, there is a "bonus" for each which adds 4 points to the new skills (making a 100 possible)

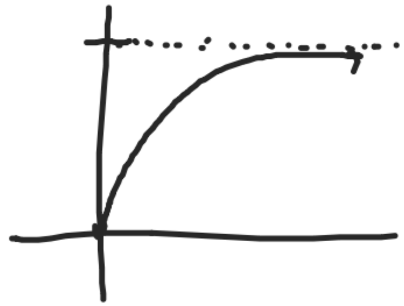
***Use extra paper if needed**

When finished, be sure your name is on it and turn into basket

Then quietly work on the AP multiple choice practice due Weds.

Please enter your PIN

2. **[P]** Imagine that you are skydiving. The graph of your speed as a function of time from the time you jumped out of the plane to the time you achieved terminal velocity is



(a) increasing and concave up

(b) decreasing and concave up

(c) increasing and concave down

(d) decreasing and concave down

(e) not sure yet

4. **[D]** Water is being poured into a "Dixie cup" (a standard cup that is smaller at the bottom than at the top). The height of the water in the cup is a function of the volume of water in the cup. The graph of this function is

(a) increasing and concave up

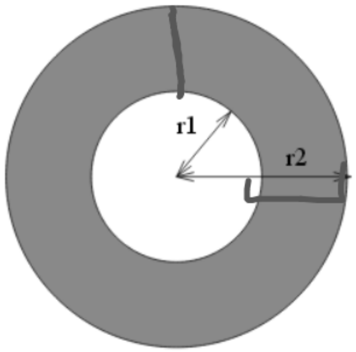
(b) increasing and concave down

(c) a straight line with positive slope.

(d) not sure yet

Answer: (b). It is easy to see that the function is increasing: the more water we add, the bigger the height. To see that the function is concave down, observe that the instantaneous rate of change of the height with respect to the volume is decreasing: as the cup gets filled, for a fixed increment in water, we get smaller and smaller increments in height.

7. **[P]** The region between two concentric circles of radius r_1 and r_2 is called an annulus. If $r_2 > r_1$, the area of the annulus is $\pi(r_2^2 - r_1^2)$.

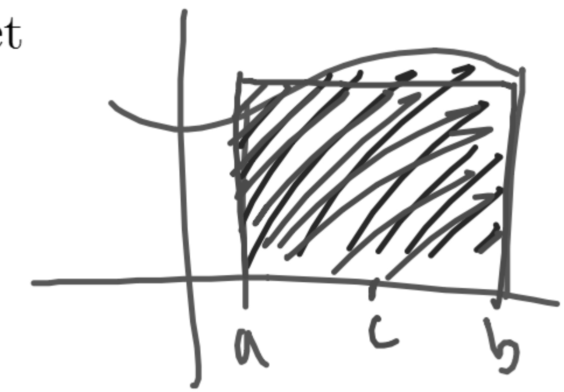


(a) This area can be approximated by a sum of areas of rectangles, but there is no single rectangle that has exactly the same area.

(b) This area cannot be approximated by the area of rectangles because the circles are concentric.

(c) There must be a radius, r, between r_1 and r_2 for which the rectangle with base $r_2 - r_1$ and height $2\pi r$ is exactly equal to the area of the annulus.

(d) not sure yet



$$f(c)(b-a) = \int_a^b f(x)\,dx$$

M.V.T.i

[Q] A racer is running back and forth along a straight path. He finishes the race at the place where he began.

True or **False**. There had to be at least one moment, other than the beginning and the end of the race, when he ”stopped” (i.e., his speed was 0). Be prepared to give a proof or counterexample.

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[D] Two racers start a race at the same moment and finish in a tie. Which of the following must be true?

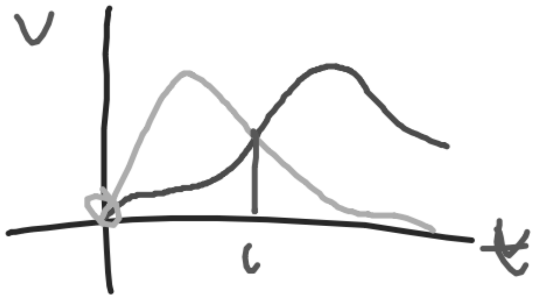
(a) At some point during the race the two racers were not tied.

(b) The racers' speeds at the end of the race must have been exactly the same.

(c) The racers must have had the same speed at exactly the same time at some point in the race.

(d) The racers had to have the same speed at some moment, but not necessarily at exactly the same time.

Answer: (c). This is a challenging problem for students. The main point here is to recall the idea (that was first introduced to them in the IVT) that we can show that two functions take the same value at a point by showing that their difference is zero. As the MVT only talks about what happens to one function, then we must look at the difference of the two functions in order to compare them at the same moment. Also discussing the problem with a graph, and showing that this happens when the slower person begins to speed up to catch up the other one will make it more clear.



[Q] If f is an antiderivative of g, and g is an antiderivative of h, then

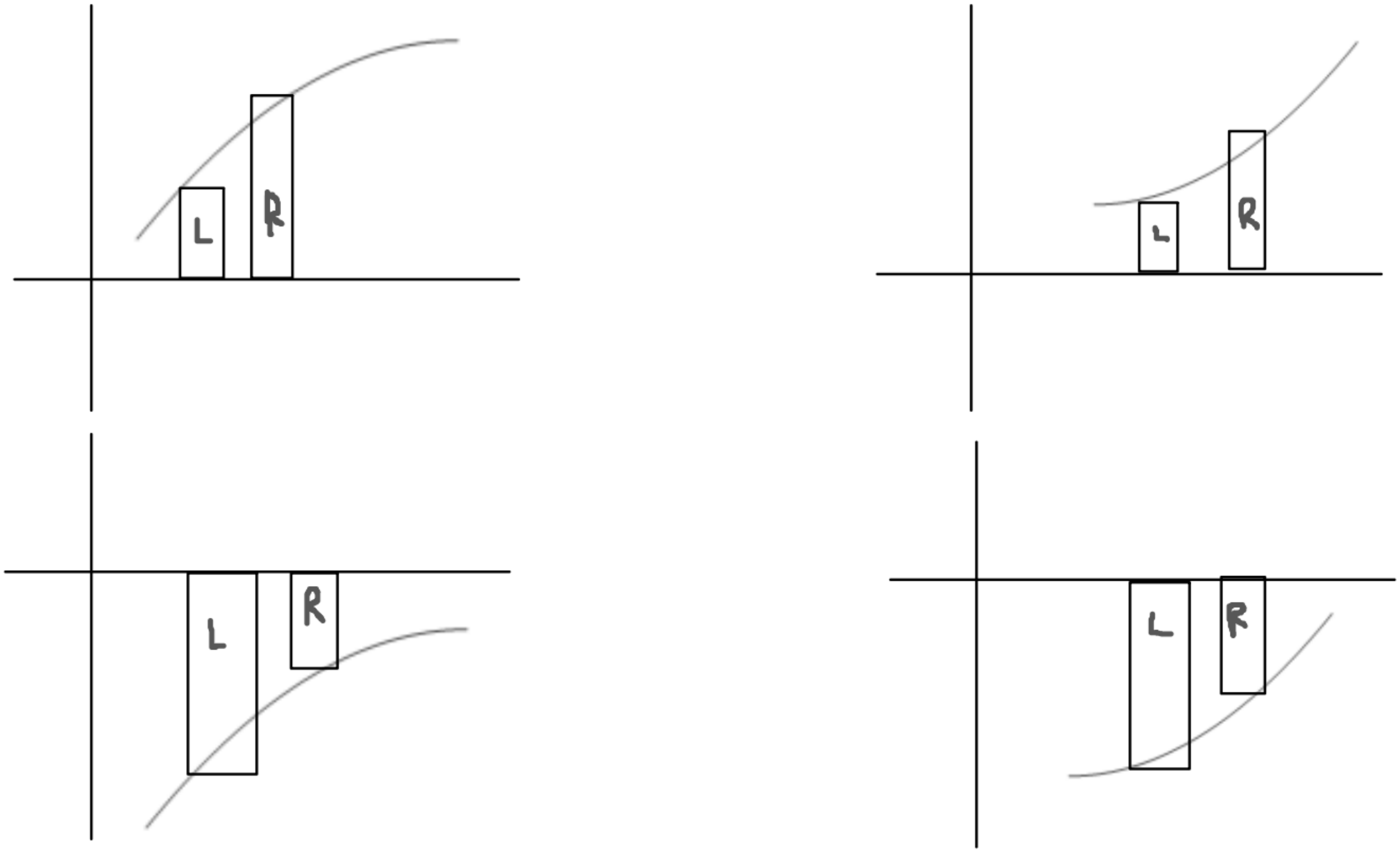
(a) h is an antiderivative of f

(b) h is the second derivative of f

(c) h is the derivative of f''

If $f(x)$ is increasing, a Riemann approximation for area yields an underestimate, then the Riemann approximation was a:

(a) Left Riemann Sum
(b) Right Riemann Sum
(c) not enough information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[P] Water is pouring out of a pipe at the rate of $f(t)$ gallons/minute. You collect the water that flows from the pipe between $t = 2$ and $t = 4$. The amount of water you collect can be represented by:

(a) $\int_2^4 f(x)dx$

(b) $f(4) - f(2)$

(c) $(4-2)f(4)$

(d) the average of $f(4)$ and $f(2)$ times the amount of time that elapsed