

Good afternoon class: Mr Mohyuddin is (sadly) out at a math meeting but Ms Bryan is here to sub today :)

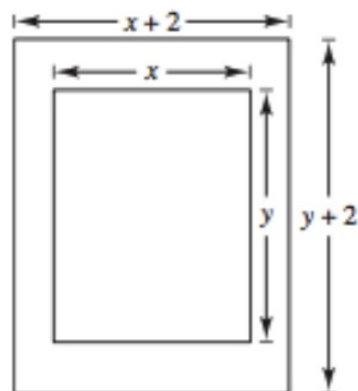
You will be receiving practice assessments for related rates shortly  
Solutions are posted at [mcalc.weebly.com](http://mcalc.weebly.com) but give them an honest attempt first!

reminders:

- tutoring is still available today after school
- no open lunch tomorrow thanks

Optimization hw p 220

17.  $x = \sqrt{30}$ , plug back in to find  $y$ ,  $y = \sqrt{30}$ . dimensions are  $2 + \sqrt{30}$



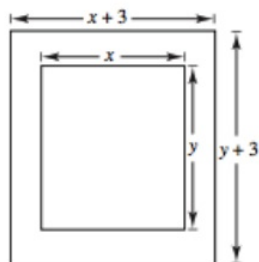
18.  $xy = 36 \Rightarrow y = \frac{36}{x}$

$$A = (x+3)(y+3) = (x+3)\left(\frac{36}{x} + 3\right)$$

$$= 36 + \frac{108}{x} + 3x + 9$$

$$\frac{dA}{dx} = \frac{-108}{x^2} + 3 = 0 \Rightarrow 3x^2 = 108 \Rightarrow x = 6, y = 6$$

Dimensions:  $9 \times 9$



19. 700 by 350

20.

$$S = 2x^2 + 4xy = 337.5$$

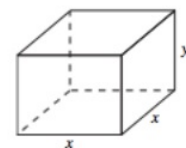
$$y = \frac{337.5 - 2x^2}{4x}$$

$$V = x^2y = x^2 \left[ \frac{337.5 - 2x^2}{4x} \right] = 84.375x - \frac{1}{2}x^3$$

$$\frac{dV}{dx} = 84.375 - \frac{3}{2}x^2 = 0 \Rightarrow x^2 = 56.25 \Rightarrow x = 7.5 \text{ and } y = 7.5.$$

$$\frac{d^2V}{dx^2} = -3x < 0 \text{ for } x = 7.5.$$

The maximum value occurs when  $x = y = 7.5$  cm.



Please work on the practice assessment for the remainder of class

If finished checking solutions online, finish up the optimization/rates

hw if needed

Good afternoon: no warm up, will take notes when bell rings

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## Indefinite Integration aka Antidifferentiation

a quick example to dissect:

$$\int x^2 dx = \frac{1}{3} x^3 + C$$

variable of integration (must match variable of integrand)  
(usually dx, just like with derivatives)

$$\int x^2 dx = \frac{1}{3} x^3 + C$$

integral sign  
(says, "here's  
a derivative...  
what was the  
original function??")

integrand  
thing we are finding  
the antiderivative of

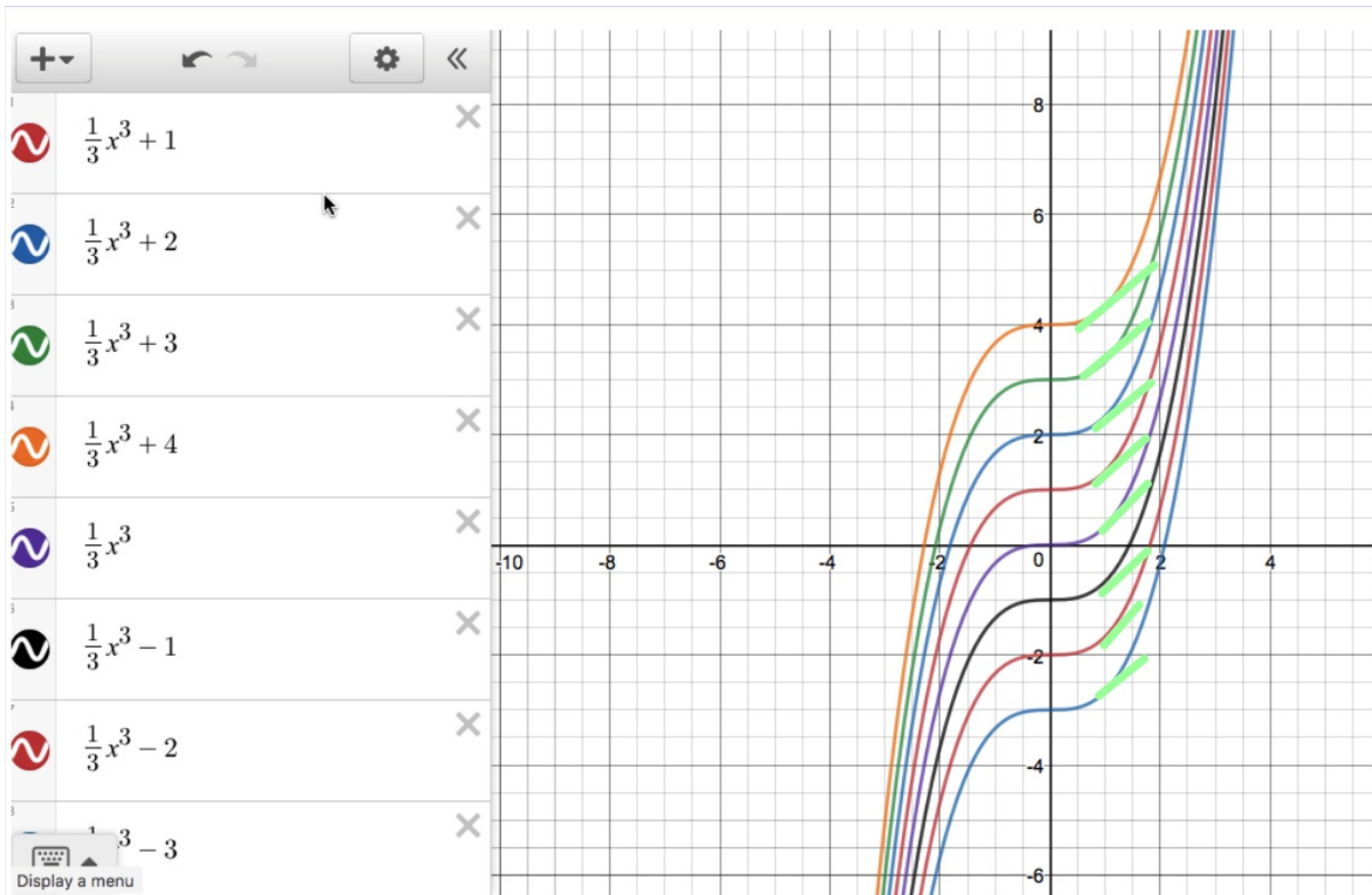
Constant of integration  
(more on this in a moment)

$$\int x^2 dx = \frac{1}{3} x^3 + C$$

The solution to an indefinite integral is a family of functions

$x^2$  is the derivative of  $\frac{1}{3} x^3$ , but also  $\frac{1}{3} x^3 + 1$ ,  $\frac{1}{3} x^3 + 4$ ,  $\frac{1}{3} x^3 - 22.1$ ....etc

Because they all have the same slopes!! adding C is just a vertical translation



Notice that all of these have the same slopes at any given x-value



Algebraically:

when you take the derivative of a constant, it goes away! so any constant could be in the original problem but disappears when given the integrand

$$\int 2x \, dx = x^2 + c$$

derivative of  $x^2 + 53$  (for example)  
is  $2x$

derivative of  $x^2 - 12$  is also  $2x$

hence, the  $+ C$

How to check if you're right??

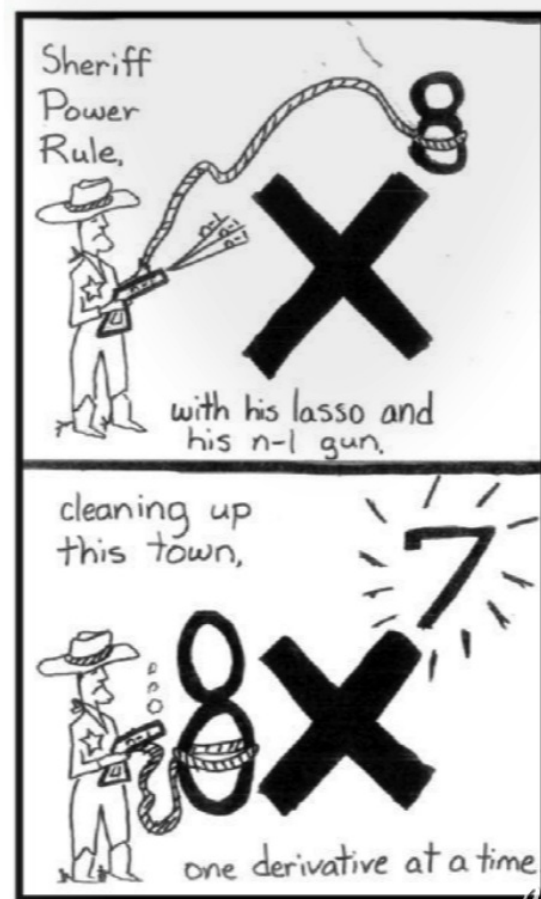
Just take the derivative of your answer and see if you get the integrand!!

$$\int \cos(x) \, dx = \sin(x) + C$$

The diagram illustrates the relationship between integration and differentiation. A red arrow labeled "integration" points from the integral of  $\cos(x)$  to the result  $\sin(x) + C$ . A blue arrow labeled "differentiation" points from the result  $\sin(x) + C$  back to the integrand  $\cos(x)$ . The result  $\sin(x) + C$  is underlined in blue.

## The Reverse Power Rule

remember this? the power rule:



$$x^n \rightarrow nx^{n-1}$$

1. multiply by exponent
2. decrement exp by 1

So in the opposite direction....

divide

- ~~1. multiply by exponent~~
- ~~2. decrement exp by 1~~

increment

So in the opposite direction....

divide

- ~~1. multiply by exponent~~
  - ~~2. decrement exp by 1~~
- increment

example:

$$\int \frac{3x^5}{6} dx$$

$$\frac{3}{6} x^6 + C$$
$$\frac{1}{2} x^6 + C$$

(ignore the 3, treat it as a coefficient as with derivs.)

check to see if it's right!!

Reverse Power Rule (Add to booklets...first ever integration formula :)

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

example in notes:

$$\int 4x^2 - 6x^4 dx$$

(recall that derivative of sum or difference is just sum/diff of the derivatives....same holds true in reverse!!)

$$\int 4x^2 - 6x^4 \, dx$$

$$4\frac{x^3}{3} - 6\frac{x^5}{5} + C$$

$$\frac{4}{3}x^3 - \frac{6}{5}x^5 + C$$

Always check to make sure it's right if ever unsure!

Lots to put into your booklets: p. 246 right column

### Differentiation Formula

$$\frac{d}{dx}[C] = 0$$

$$\frac{d}{dx}[kx] = k$$

$$\frac{d}{dx}[kf(x)] = kf'(x)$$

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

### Integration Formula

$$\int 0 \, dx = C$$

$$\int k \, dx = kx + C$$

$$\int kf(x) \, dx = k \int f(x) \, dx$$

$$\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \quad \text{Power Rule}$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

ADD THESE



example:

$$\int \frac{2}{\sqrt[3]{x}} dx$$



$$\int \frac{2}{\sqrt[3]{x}} dx$$

rrrrrrreemix

$$\int \frac{2}{x^{1/3}} dx$$

$$\int 2x^{-1/3} dx$$

$$2 \frac{x^{2/3}}{2/3} + C$$

$$\frac{\cancel{2}}{1} \cdot \frac{\cancel{3}}{2} x^{2/3} + C \rightarrow 3x^{2/3} + C$$

Dividing fractions:

$$\frac{a}{\frac{b}{c}} \rightarrow \frac{a}{1} \cdot \frac{c}{b} \rightarrow \frac{ac}{b}$$

"keep it, flip it, change it"

see p248 for more examples

classwork/homework

- p 251: #7-26, 50

- practice assessment on related rates

