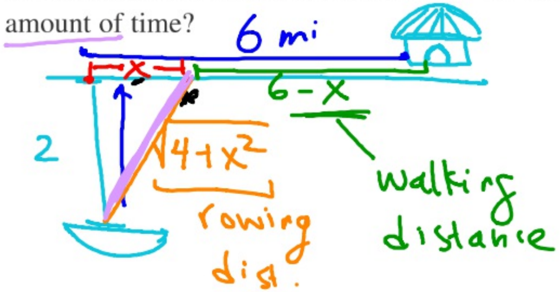
Good afternoon: the test has been converted to a take-home test

We will randomize and practice both skills a bit more before starting antidifferentiation

Take home tests due at the start of class on Thursday

Quickest Route Jane is 2 mi offshore in a boat and wishes to reach a coastal village 6 mi down a straight shoreline from the point nearest the boat. She can row 2 mph and can walk 5 mph. Where should she land her boat to reach the village in the least amount of time?



Constraint:

row: $\sqrt{4+x^2}$

Optimize:

$$T = \frac{1}{2}\sqrt{4+x^2} + \frac{1}{5}(6-x)$$

row time, walk time

$$T = \frac{1}{2}(4+x^2)^{1/2} + \frac{6}{5} - \frac{1}{5}x$$

Find C.N.

$$\frac{dT}{dx} = \frac{1}{4}(4+x^2)^{-1/2}\cdot 2x - \frac{1}{5} = 0$$

$$\frac{2x}{4(4+x^2)^{1/2}} = \frac{1}{5}$$

$$10x = 4(4+x^2)^{1/2}$$

$$5x = 2(4+x^2)^{1/2}$$

$$25x^2 = 4(4+x^2)$$

$$25x^2 = 16 + 4x^2$$

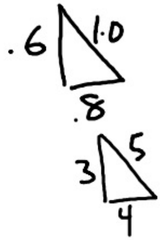
$$21x^2 = 16$$

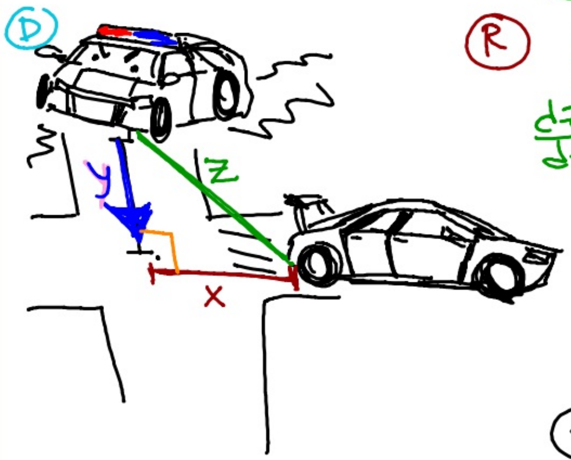
$$x^2 = \frac{16}{21} \rightarrow x = \frac{4}{\sqrt{21}} \approx 0.873$$

$$x = \frac{4}{\sqrt{21}}$$

$$\sqrt{A^2+B^2} \neq A+B$$

A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?





(R) $\frac{dx}{dt} = ?$ $\quad \frac{dy}{dt} = -60$

$\frac{dz}{dt} = 20$ $\quad x = 0.8,\ y = 0.6 \rightsquigarrow z = 1$

(E) $x^2 + y^2 = z^2$

(D) $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

(S) $x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$

$\frac{8}{10}\cdot\frac{dx}{dt} + \frac{6}{10}\cdot -60 = 1\cdot 20$

$\frac{8}{10}\cdot\frac{dx}{dt} - 36 = 20$

$\frac{dx}{dt} = (56)\frac{10}{8}$

$\frac{dx}{dt} = 70$

Now for something different

Suppose some unknown function has derivative
$f'(x) = 4x^3 - 2x^2 + 3x - 6$

What could have been the original function?

$$x^4 - \frac{2}{3}x^3 + \frac{3}{2}x^2 - 6x + C$$

(some constant)

Indefinite Integration aka Antidifferentiation

a quick example to dissect:

$$\int x^2 \quad dx \qquad = \quad \frac{1}{3} \; x^3 \quad + \; C$$

variable of integration (must match variable of integrand)

(usually dx, just like with derivatives)

$$\int x^2 \, dx = \frac{1}{3} x^3 + C$$

Answer
"antiderivative"

"Summation"

<u>integral sign</u>
(says, "here's a derivative... what was the original function??")

<u>integrand</u>
thing we are finding the antiderivative of

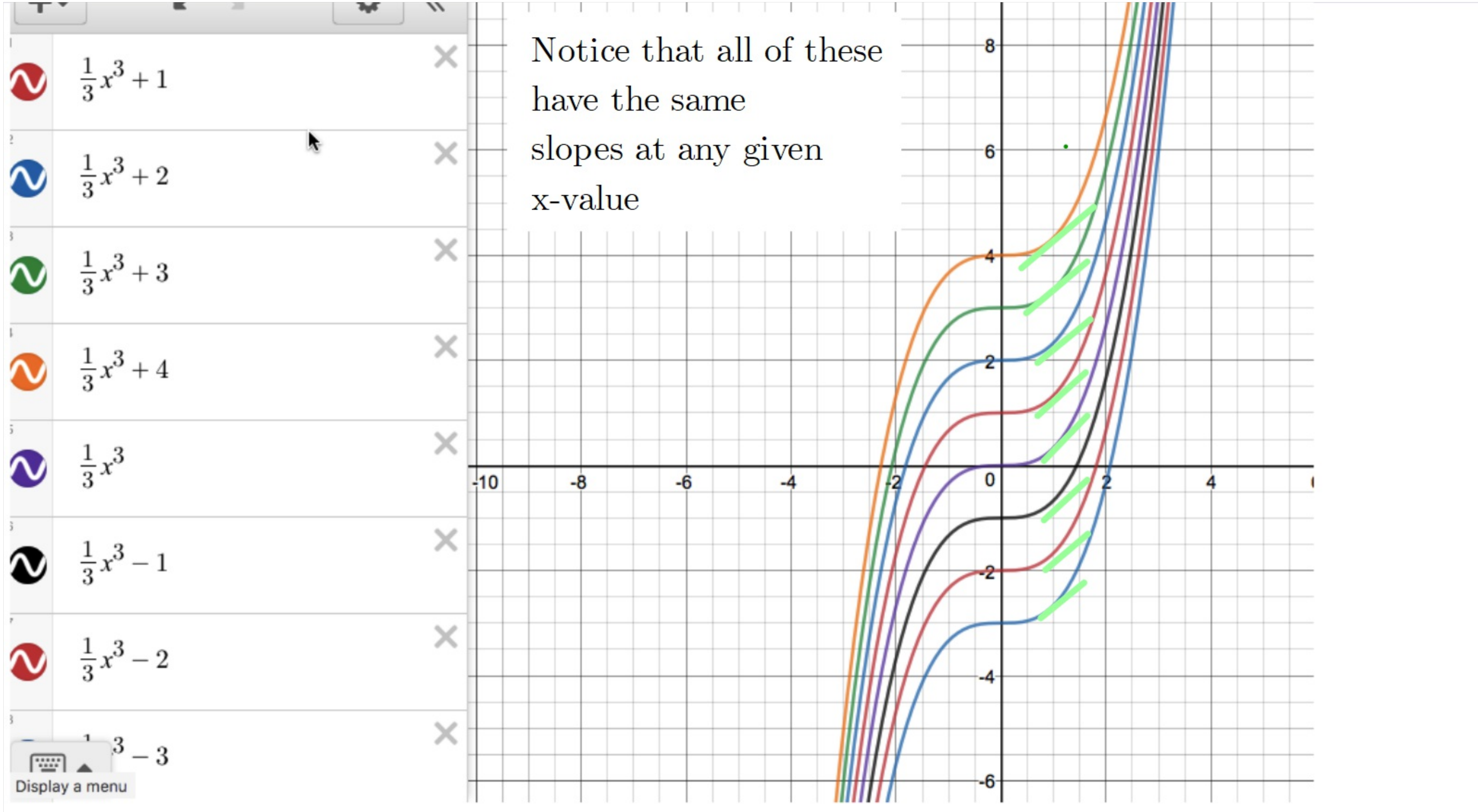
<u>Constant of integration</u>
(more on this in a moment)

$$\int x^2 \, dx = \frac{1}{3} x^3 + C$$

The solution to an indefinite integral is a family of functions

x^2 is the derivative of $\frac{1}{3}x^3$, but also $\frac{1}{3}x^3 + 1$, $\frac{1}{3}x^3 + 4$, $\frac{1}{3}x^3$ -22.1....etc

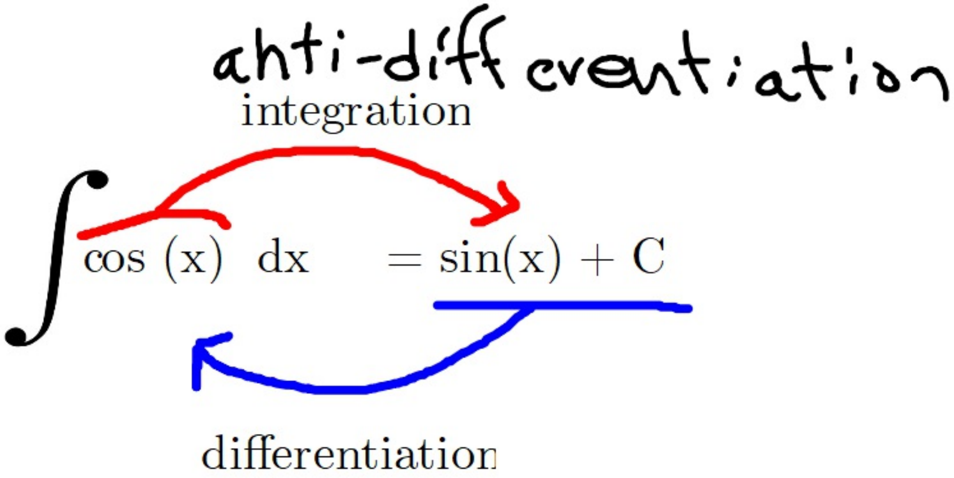
Because they all have the same slopes!! adding C is just a vertical translation



$\frac{1}{3}x^3+1$
$\frac{1}{3}x^3+2$
$\frac{1}{3}x^3+3$
$\frac{1}{3}x^3+4$
$\frac{1}{3}x^3$
$\frac{1}{3}x^3-1$
$\frac{1}{3}x^3-2$
$\frac{1}{3}x^3-3$
Display a menu
Notice that all of these have the same slopes at any given x-value
8
6
4
2
0
-2
-4
-6
-10
-8
-6
-4
-2
2
4

How to check if you're right??

Just take the derivative of your answer and see if you get the integrand!!



The Reverse Power Rule

remember this? the power rule:

$x^n \rightarrow nx^{n-1}$

1. multiply by exponent
2. decrement exp by 1

$\frac{d}{dx}$ Power Rule

$x^n \rightarrow nx^{n-1}$

1. multiply by exponent
2. decrement exp by 1

$\int$ Reverse Power Rule

So in the opposite direction....

1. ~~multiply~~ divide by exponent
2. ~~decrement~~ increment exp by 1

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

or, $\frac{1}{n+1} \cdot x^{n+1} + C$

example:

$$\int 3x^5 \, dx$$

$$\frac{3x^6}{6} + C$$

$$\frac{1}{2}x^6 + C$$

$$Ax^n$$

$$\int -\frac{2}{5}x^6 + 21\sqrt{x}\ dx$$

$$\int -\frac{2}{5}x^6 + 21x^{\frac{1}{2}}\ dx$$

$$-\frac{2}{5}\cdot\frac{1}{7}x^7 + 21\cdot\frac{2}{3}x^{\frac{3}{2}}$$

$$-\frac{2}{35}x^7 + 14x^{3/2} + C$$

$$\int 4x^2 - 6x^4\ \ dx$$

$$\frac{4}{3}x^3 - \frac{6}{5}x^5 + C$$

$$\int \frac{2}{\sqrt[3]{x}}\,dx$$

$$\int 2x^{-1/3}\,dx$$

$$2\cdot\frac{x^{2/3}}{2/3}+C$$

$$2\cdot\frac{3}{2}x^{2/3}+C$$

$$3x^{2/3}+C$$

$$\int \frac{5}{x}\,dx$$

$$\int 5x^{-1}\,dx$$

$$\frac{5x^{0}}{0}+C$$

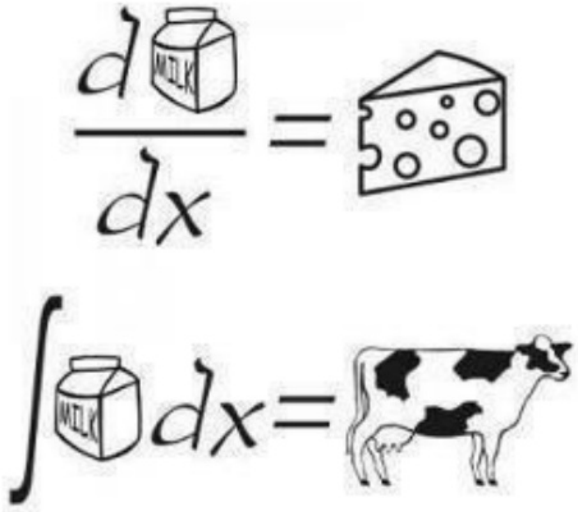
puh?? guh??

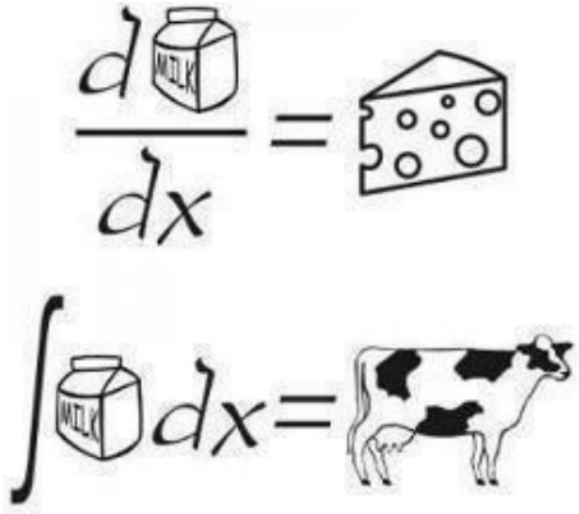
$$5\int \frac{1}{x}\,dx$$

$$5\ln|x|+C$$

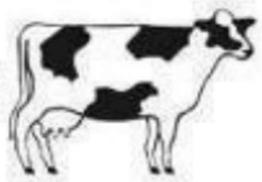
$$\int \frac{5}{x} dx$$

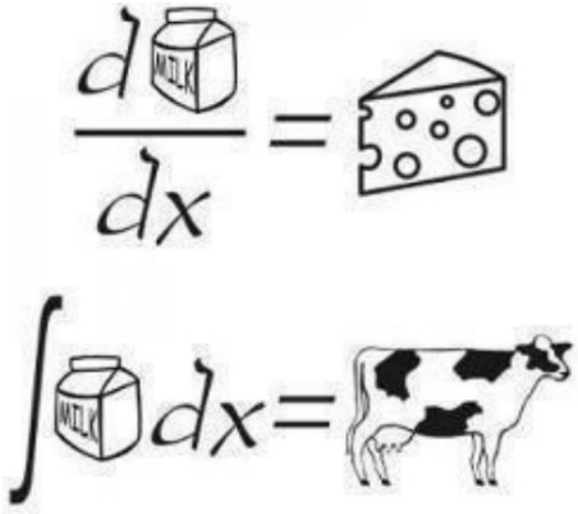
$$\int e^{3x}\, dx$$


d MILK
dx
MILK dx



What is

$\frac{d}{dx}$  ?



What is

∫  dx ?

Take home test due at start of class Thursday

Can work together, use resources,
use calc (show all steps on paper)

But must do own work