

Another Big Ol Practice Assessment

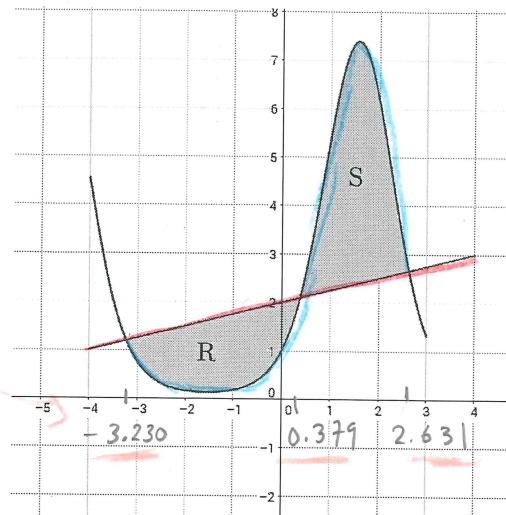
1. Let $f(x) = e^{2 \sin x}$ and $g(x) = \frac{1}{4}x + 2$ be the boundaries of the regions R and S . Find the total area of R and S .

use 2ND → TRACE → INTERSECT

$$\int_{-3.23}^{0.379} \left(\frac{1}{4}x + 2 \right) - (e^{2 \sin x}) dx \quad \begin{matrix} \text{Math} \\ \text{→ 4.196} \end{matrix}$$

top = bottom

$$\int_{0.379}^{2.631} (e^{2 \sin x}) - \left(\frac{1}{4}x + 2 \right) dx \quad \begin{matrix} \text{Math} \\ \text{→ 6.457} \end{matrix}$$



$$10.653 \text{ u}^2$$

I-U7: Given $\int_0^5 f(x) dx = 10$

$$\int_5^7 f(x) dx = 3$$

$$\int_{-2}^5 f(x) dx = -2$$

Find each of the following:

2. $\int_7^{-2} f(x) dx$

$$\int_7^{-2} = - \left[\int_{-2}^7 \right] \rightarrow - \left[\int_{-2}^5 + \int_5^7 \right] - \left[-2 + 3 \right] \rightarrow - [1] = (-1)$$

3. $\int_0^{-2} f(x) dx$

$$\int_0^{-2} = \int_0^5 + \int_5^{-2} \rightarrow \int_0^5 - \int_5^{-2} \quad 10 - -2 \rightarrow 12$$

I-U4

Let $f(x) = \int_3^{2x} 2t^2 - 3t + 2 dt$.

4. Find $f'(x)$. Simplify your answer.

FTC $f(x) = \int_3^{2x} 2t^2 - 3t + 2 dt$

$$f'(x) = (2(2x)^2 - 3(2x) + 2) \cdot 2 \quad \begin{matrix} \text{chain} \\ \text{rule} \end{matrix} \quad \begin{matrix} (2(4x^2) - 6x + 2) \cdot 2 \\ 16x^2 - 12x + 4 = f'(x) \end{matrix}$$

5. Find all intervals where $f(x)$ is increasing. Justify your answer.

$$f'(x) > 0$$

From #4:

$$f'(x) = 16x^2 - 12x + 4 = 0$$

n.? $4(4x^2 - 3x + 1) = 0$

$$4(4x+1)(x-1) = 0$$

$$\begin{matrix} \downarrow \\ x = -\frac{1}{4} \end{matrix} \quad x = 1 \quad \begin{matrix} \text{C.N.} \end{matrix}$$

f is increasing on

$(-\infty, -\frac{1}{4})$ and
 $(1, \infty)$ b/c

f' is positive
here

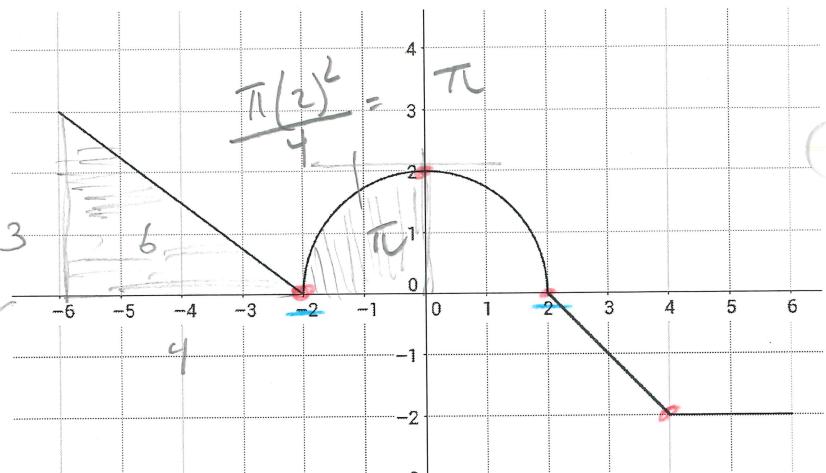
$$\begin{array}{c|ccc|c} & f' & + & - & + \\ \hline x & & & & \\ \hline -\frac{1}{4} & & & & \\ & & & & \\ 1 & & & & \end{array}$$

Plug values into f'

$$\begin{aligned} f'(-\infty) &= + \\ f'(0) &= - \\ f'(1) &= + \end{aligned}$$

I-U9

The function $f(t)$ is shown over $[-6, 6]$ and consists of line segments and a semicircle. Let $G(x) = \int_{-6}^x f(t) dt$



6. Find $G(0)$, $G'(0)$, and $G''(0)$.

$$\begin{array}{|c|} \hline \text{FTC} \\ G = \int f \\ \hline \end{array}$$

$$G(0) = \int_{-6}^0 f(t) dt$$

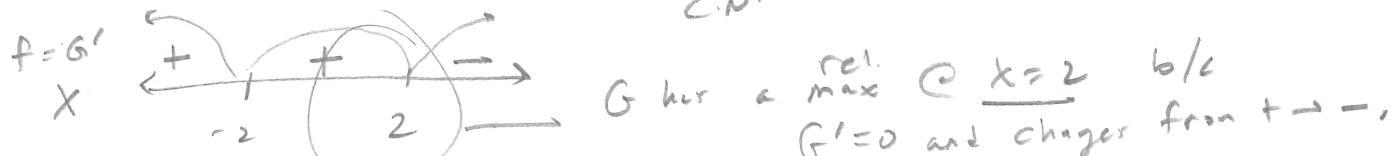
$$G'(0) = f(0) = 2$$

$$G''(0) = f'(0) = \text{slope of } f \text{ at } x=0 \rightarrow 0$$

7. Find the relative maxima of $G(x)$, if any, over $[-6, 6]$. Justify your answer.

$$G' = 0, + \rightarrow -$$

$$G' = 0 \xrightarrow{\text{FTC}} f = 0 \rightarrow b, g, h: \underline{-2, 2} \text{ C.N.}$$



8. Find any points of inflection of $G(x)$. Justify your answer.

$$G'' = 0, \text{ sign change}$$

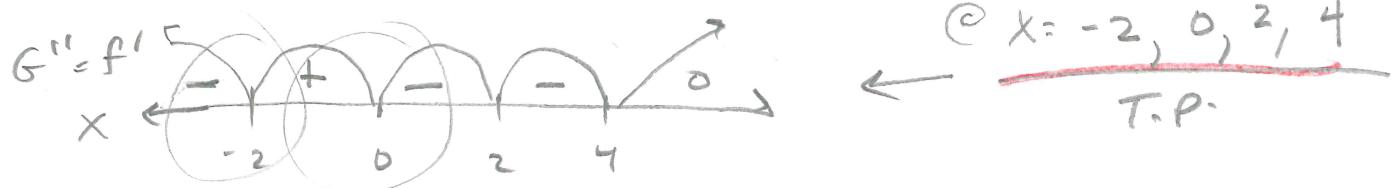
or
undef.

$$G(2) = 6 + 2\pi$$

Max. Value

$$G'' = 0 \xrightarrow{\text{FTC}} f' = 0 \Rightarrow \text{slope of } f = 0 \text{ by graph}$$

(undef) (undef) (undef)



G has inflection pt @ $x = -2$ and $x = 0$

b/c $G'' = 0$ and G'' changes sign.

I-A7b

$$\text{Net change: } f(b) = f(a) + \int_a^b f'(t) dt$$

$$f = 10 \quad t = 24$$

9. It's 10am and Frank has already used 8 mb of data on his cell phone. From 10am to midnight ($t=24$), his data usage rate can be modeled by the differentiable function $f(t) = \sin\left(\frac{\pi}{8}t\right) + 1$ mb/hr. Write an equation that includes an integral that will give the amount of data Frank has used as of midnight. Then, find that amount and include units in your answer.

$F(x) = 8 + \int_{10}^x \sin\left(\frac{\pi}{8}t\right) + 1 dt$

↑
time after
10 am

↑
 $F(10)$

accumulation of rate

$$F(24) = 8 + \int_{10}^{24} \sin\left(\frac{\pi}{8}t\right) + 1 dt$$

 Math - 9

$$= 8 + 14.746$$

$$22.746 \text{ mb}$$

I-A7a

10. Find the average value of $f(x) = \frac{1}{x}$ over the interval $[1, 3]$

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$\frac{1}{3-1} \int_1^3 \frac{1}{x} dx$$

$$\frac{1}{2} [\ln|x|]_1^3 \rightarrow \frac{1}{2} [\ln 3 - \ln 1]$$

$$\frac{1}{2} [\ln 3 - 0] \rightarrow \frac{1}{2} \ln 3 \rightarrow \ln \sqrt{3}$$

11. Let $Q'(t) = 1 - \cos\left(\frac{\pi t}{5}\right)$ model the rate, in hundreds of people per hour, entering an amusement park. Using correct units, explain the meaning of $\frac{1}{5} \int_2^7 Q'(t) dt$ in context. Then, find its value.

average rate, in number of people per hour, of people entering park over hours $t=2$ to $t=7$.

$$= \frac{1}{5} \int_2^7 (1 - \cos\left(\frac{\pi}{5}t\right)) dt$$

 Math - 9

$$= \frac{1}{5} (8.027)$$

$$= 1.605 \rightarrow 160 \text{ ppl/hr}$$

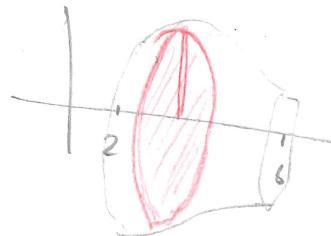
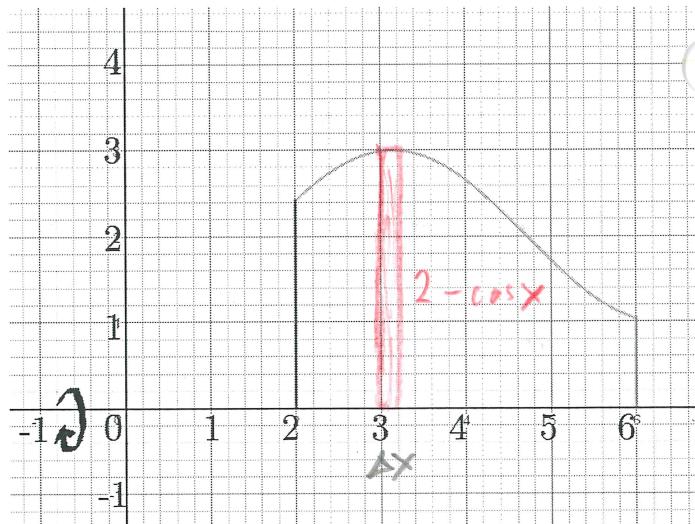
I-A5a

12. Find the volume of the solid generated by revolving the region bounded by $f(x) = 2 - \cos x$ and the vertical lines $x = 2$ and $x = 6$ about the x-axis. Show all work.

$$V = \pi \int_2^6 (F(x))^2 dx$$

$$= \pi \int_2^6 (2 - \cos x)^2 dx$$

$$22.81\pi u^3$$



13. Set-up a single integral to calculate the volume of the solid generated when the region bounded by $f(x) = x^2 - 2x$ and $g(x) = x$ is revolved around the axis $y = 4$. Then use a calculator to find that volume.

$$V = \pi \int_0^3 ((\text{outer rad})^2 - (\text{inner rad})^2) dx$$

(top - bottom) (top - bottom)

$$V = \pi \int_0^3 (4 - (x^2 - 2x))^2 - (4 - x)^2 dx$$

$$V = \pi \int_0^3 (4 - x^2 + 2x)^2 - (4 - x)^2 dx$$

$\boxed{\text{math-9}}$

$$\pi \cdot 30.6$$

$$30.6\pi u^3$$

