I-U3a Practice Assessment

1. *Find the left Riemann sum approximation of* $\int_1^4 -(x-3)^2+5\,dx$ *using 6 intervals of equal length. Use 3 decimal places of accuracy.*

Solution

$\Delta x = \dfrac{b-a}{n} = \dfrac{4-1}{6} = \dfrac{3}{6} = \dfrac{1}{2}$ *so intervals are spaced by* $\dfrac{1}{2}$, *being* 1, 1.5, 2, 2.5, 3, 3.5, *and* 4

Since we want left sum, we use the all but the last one: 1, 1.5, 2, 2.5, 3, 3.5. Notice that this is 6 values , as required.

Riemann sum:

$$\sum_{i=1}^{6} f(x_i)dx = f(1)\left(\frac{1}{2}\right) + f(1.5)\left(\frac{1}{2}\right) + f(2)\left(\frac{1}{2}\right) + f(2.5)\left(\frac{1}{2}\right) + f(3)\left(\frac{1}{2}\right) + f(3.5)\left(\frac{1}{2}\right)$$

Factor out a $\frac{1}{2}$ $\quad \frac{1}{2}[f(1)+f(1.5)+f(2)+f(2.5)+f(3)+f(3.5)]$

Plug each x into f(x) $\quad \frac{1}{2}[1+2.75+4+4.75+5+4.75]$

$\frac{1}{2}[22.25] = 11.125$

2. *Is your answer in #1 an over or under approximation? Explain.*

Solution

Since this function is increasing, the left-endpoint's y-value is always <u>less</u> than the right endpoint's y-value. Thus the rectangles are shorter, so it is an under approximation.

3. *Approximate* $\int_{-5}^{-1} -\frac{5}{x}dx$ *using a right Riemann approximation using 4 intervals of equal length. Use 3 decimal places of accuracy.*

Solution

$\Delta x = \dfrac{b-a}{n} = \dfrac{-1--5}{4} = \dfrac{4}{4} = 1$ *so intervals are spaced by* 1 *being* $-5, -4, -3, -2,$ *and* -1

Since we want Right sum, we use the all but the first one: -4,-3,-2,-1. Notice that this is 4 values , as required.

Riemann sum:

$$\sum_{i=1}^{4} f(x_i)dx = f(-4)(1) + f(-3)(1) + f(-2)(1) + f(-1)(1)$$

Factor out a 1 $\quad 1[f(-4)+f(-3)+f(-2)+f(-1)]$

Plug each x into f(x) $\quad 1[1.25+1.667+2.5+5]$

$1[10.417] = 10.471$

4. *Is your answer in #3 an over or under approximation? Explain.*

Solution

Since this function is increasing, the right-endpoint's y-value is always <u>more</u> than the left endpoint's y-value. Thus the rectangles are taller, so it is an over-approximation.