HW sols p.453:

3. $V = \pi \int_1^4 \left(\sqrt{x}\right)^2 dx = \pi \int_1^4 x\, dx = \pi \left[\frac{x^2}{2}\right]_1^4 = \frac{15\pi}{2}$

4. $V = \pi \int_0^3 \left(\sqrt{9 - x^2}\right)^2 dx = \pi \int_0^3 \left(9 - x^2\right) dx = \pi \left[9x - \frac{x^3}{3}\right]_0^3 = 18\pi$

$$V = \int_a^b \pi \left(f(x)\right)^2 dx$$

7. $y = x^2 \Rightarrow x = \sqrt{y}$

$$V = \pi \int_0^4 \left(\sqrt{y}\right)^2 dy = \pi \int_0^4 y\, dy$$

$$= \pi \left[\frac{y^2}{2}\right]_0^4 = 8\pi$$

8. $y = \sqrt{16 - x^2} \Rightarrow x = \sqrt{16 - y^2}$

$$V = \pi \int_0^4 \left(\sqrt{16 - y^2}\right)^2 dy = \pi \int_0^4 \left(16 - y^2\right) dy$$

$$= \pi \left[16y - \frac{y^3}{3}\right]_0^4 = \frac{128\pi}{3}$$

**[Q]** A sprinter practices by running various distances back and forth in a straight line in a gym. Her velocity at $t$ seconds is given by the function $v(t)$. What does $\int_0^{60} |v(t)| dt$ represent?

(a) The total distance the sprinter ran in one minute

(b) The sprinter's average velocity in one minute

(c) The sprinter's distance from the starting point after one minute

(d) None of the above

[**P**] Suppose $f$ is a differentiable function. Then $\int_0^x f'(t)\,dt = f(x) - f(0)$

$$\int_0^{\pi} \cos(x)\,dx = \Big[\sin(x)\Big]_0^{\pi}$$

(a) Always

(b) Sometimes

(c) Never

Justify your answer.

[**P**] Suppose the function $f(t)$ is continuous and always positive. If $G$ is an antiderivative of $f$, then we know that $G$:

$G' = f > 0$

$f > 0$

(a) is always positive.

(b) is sometimes positive and sometimes negative.

(c) is always increasing.

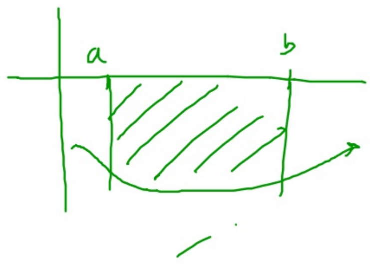
(d) There is not enough information to conclude any of the above.

[**Q**] If $f$ is continuous and $f(x) < 0$ for all $x \in [a, b]$, then $\int_a^b f(x)dx$

(a) must be negative

(b) might be 0

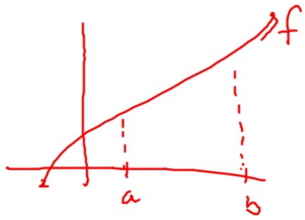
(c) not enough information



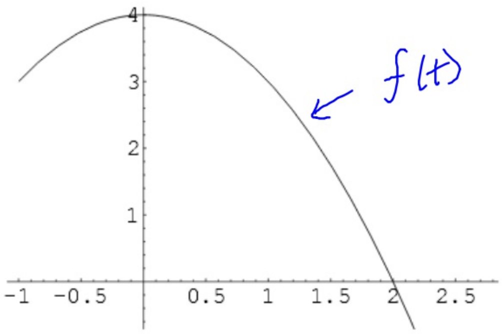
**True** or **False**. If $f$ is continuous on the interval $[a, b]$, $\frac{d}{dx}\left(\boxed{\int_a^b f(x)dx}\right) = f(x)$.

$\neq$

0




$$\frac{d}{dx}\int_a^x f(t)\,dt$$




$g = \int f$

$g' = f \leftarrow$ pos

$g'' = f'$

Let $g(x) = \displaystyle\int_0^x f(t)\,dt$. Then for $0 < x < 2$, $g(x)$ is

(a) increasing and concave up.

(b) increasing and concave down.

~~(c)~~ decreasing and concave up.

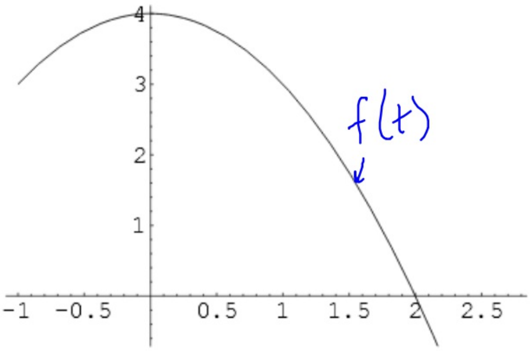
~~(d)~~ decreasing and concave down.

$g = \int f$

$g' = f$

$g'(0) = f(0)$

$f(2) =$




Let $g(x) = \int_0^x f(t)\,dt$. Then

(a) $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 0$

(b) $g(0) = 0$, $g'(0) = 4$ and $g'(2) = 0$

~~(c)~~ $g(0) = 1$, $g'(0) = 0$ and $g'(2) = 1$

(d) $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 1$

**[P]** You are traveling with velocity $v(t)$ that varies continuously over the interval $[a, b]$ and your position at time $t$ is given by $s(t)$. Which of the following represent your average velocity for that time interval:

(I) $\dfrac{\int_a^b v(t)dt}{(b-a)}$ $\quad \dfrac{1}{b-a}\int_a^b v(t)\,dt$

(II) $\dfrac{s(b)-s(a)}{b-a}$ $\Big\}$ slope = avg. r.o.c.

(III) $v(c)$ for at least one $c$ between $a$ and $b$

(a) I, II, and III

(b) I only

(c) I and II only