[Group Work: Final 30 minutes of class]

Agenda:

- Average rate examples
- Mean Value Theorem for Integrals
- Using accumulation

NOTES

Find the average value of $f(x) = \dfrac{4(x^2+1)}{x^2}$ over the interval [1,3]

avg val $= \dfrac{1}{b-a} \cdot \int_a^b f(x)\,dx$

$$\frac{1}{3-1}\int_1^3 \frac{4x^2+4}{x^2}\,dx$$

$$\frac{1}{2}\int_1^3 4 + \frac{4}{x^2}\,dx$$

$$\frac{1}{2}\int_1^3 4 + 4x^{-2}\,dx$$

$$\int 4x^{-2}$$

$$4\,\frac{x^{-1}}{-1}$$

$$-4x^{-1}$$

$$\boxed{x^{-n} \Leftrightarrow \frac{1}{x^n}}$$

$$\frac{1}{2}\left[4x - \frac{4}{x}\right]_1^3$$

$$\frac{1}{2}\left[\left(\cancel{4\cdot 3}^{\frac{36}{3}} - \frac{4}{3}\right) - \cancel{\left(4\cdot 1 - \frac{4}{1}\right)}^{0}\right]$$

$$\frac{1}{\cancel{2}}\;\frac{\cancel{32}\,16}{3} \approx \underline{5.333}$$

What is the mean of 95, 98, 92, 90, 85?

92

Discrete

vs Continuous

[redacted]

What is the mean of 87, 91, 95, 100, 100?

94.6

Continuous sets must contain their mean

So,

Somewhere within [1,3], f(x) equals exactly 16/3

where is $f(x) = \frac{16}{3}$?

$$\frac{4(x^2+1)}{x^2} = \frac{16}{3}$$

$$16x^2 = 12(x^2+1)$$

$$16x^2 = 12x^2+12$$

$$4x^2-12=0$$

$$4(x^2-3)=0$$

$$4(x-\sqrt{3})(x+\sqrt{3})=0$$

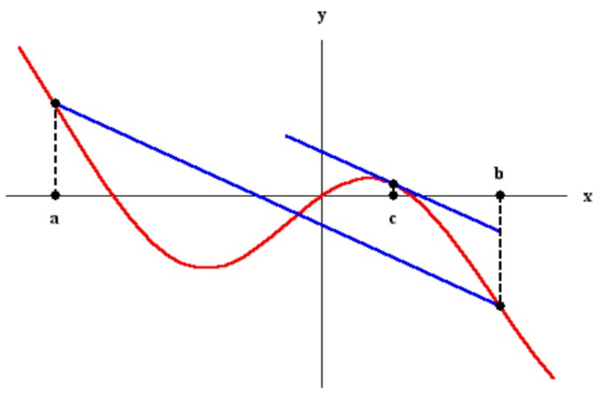
$$x=\pm\sqrt{3} \longrightarrow \boxed{x=\sqrt{3}}$$

$4x^2=12$

$x^2=3$

$x=\pm\sqrt{3}$

Review: Mean Value Theorem



For g(x) differentiable on (a,b) there exists some c in (a,b) such that

$$g'(x) = \frac{g(b)-g(a)}{b-a}$$

instant slope

average slope

Mean Value Theorem for Integrals (NEW!)

$$g'(x) = \frac{g(b)-g(a)}{b-a}$$

$$g'(x) = \frac{1}{b-a} \cdot \boxed{\left[g(b) - g(a)\right]}$$

$$\Downarrow$$

$$\underbrace{g'(x) = \frac{1}{b-a} \int_a^b g'(x)\,dx}_{\text{MV.T for Integral.}}$$

F.T.C. sec...

$$\int_a^b g'(x)\,dx = g(b) - g(a)$$

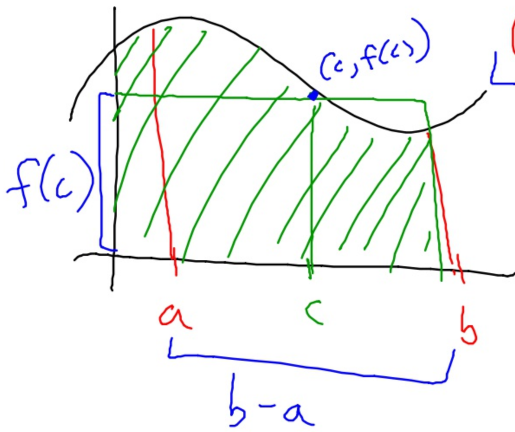
ex $\int_1^2 2x\,dx = \left[x^2\right]_1^2$

$2^2 - 1^2$

Add to ~~notes~~ + booklet

~~back to Notes~~

$$\underbrace{f(c)}_{\text{instantaneous value}} = \underbrace{\frac{1}{b-a}\int_a^b f(x)\,dx}_{\text{average value}}$$



$$\underbrace{(b-a)f(c)}_{\text{base} \times \text{height}} = \underbrace{\int_a^b f(x)\,dx}_{\text{area under curve}}$$

Area of 1 big rectangle

Net Change

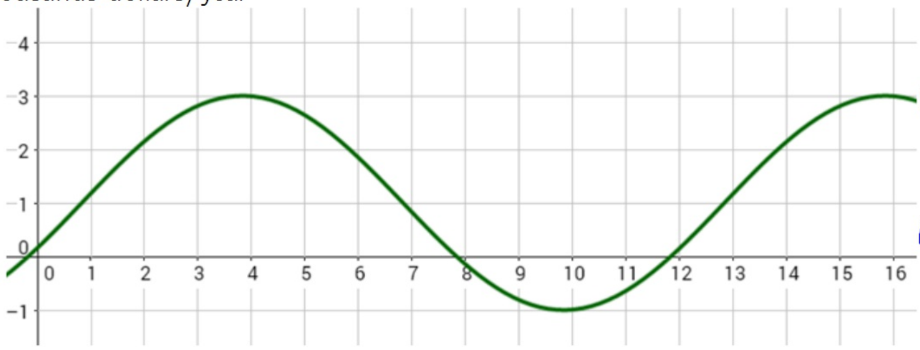
Let $R'(t)$ model the rate at which our retirement fund earns money in thousands of dollars per year after 2000 ($t=0$).

$$R'(t) = 2\sin\left(\frac{\pi t}{6} - 13\right) + 1$$

In 2000, the fund had $25,000 in it. How much money is in the fund in 2008? In 2016?

R'(t),
thousands dollars/year

$$R'(t) = 2\sin\left(\frac{\pi t}{6} -13\right) + 1$$



t, years

FTC says that:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

$$\underbrace{f(b) = f(a) + \int_a^b f'(x)\,dx}_{\text{Net Change theorem}}$$

$R(0) = 25$

$R(8) = ?$

Net Change Theorem

FTC2 says this:

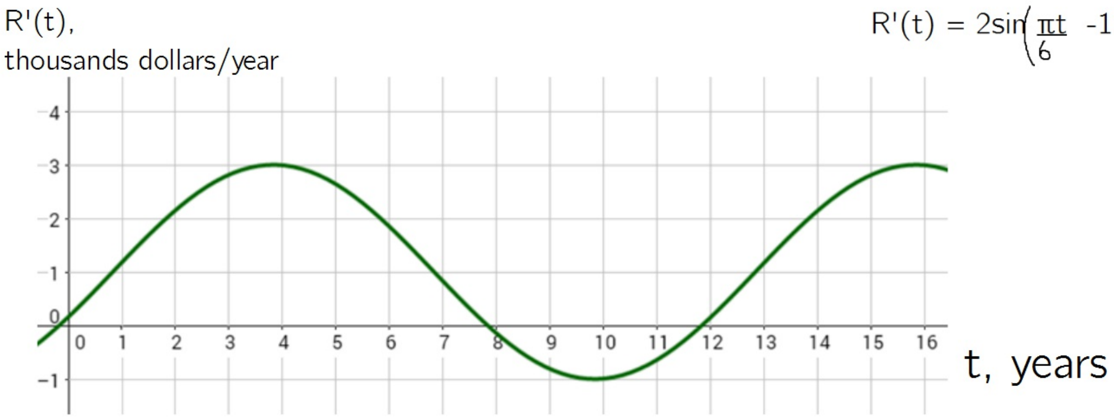
$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

Rearrange so that it becomes the net change theorem

f(b) = f(a) + (integral from a to b of f ' (x) dx)

future value at b = starting value at a + sum of all the incremental changes from a to b

To Be Continued



$$R'(t) = 2\sin\left(\frac{\pi t}{6} - 13\right) + 1$$

$R(0) = 25$

$R(8) = ?$

Homework due Friday

Worksheet #1-14

p. 288: #45-55 odd [I-A7a]