Good afternoon: if you still have your related rates take home assessment, please turn those in ASAP

We will check over hw in a few and take some more notes on reverse chain rule

More with the Reverse Chain Rule

$$\frac{1}{4}\int 4\cdot 3x^2\cdot\cos(4x^3)\,dx$$

$$\underbrace{\frac{1}{4}\int 12x^2\cdot\cos(4x^3)\,dx}$$ $= \sin$

$$\frac{1}{4}\left[\sin(4x^3)+C\right]$$

$$\frac{1}{4}\sin(4x^3)+c$$

$$\int \sin x\,dx$$

$$-\cos x + C$$

$$\frac{3}{10}\int \frac{10}{3}\, 3x \cdot e^{5x^2+1}\, dx$$

$$\boxed{\frac{3}{10} e^{5x^2+1} + C}$$

$$\int \frac{5x^2}{4x^3+2}\,dx$$

$$\frac{5}{12}\int \frac{12}{5}\,5x^2 \cdot \frac{1}{4x^3+2}\,dx$$

$$\frac{5}{12}\int 12x^2 \cdot \frac{1}{4x^3+2}\,dx$$

$$\frac{5}{12}\left[\ln\left|4x^3+2\right| + C\right]$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Good afternoon: warm up in your notebooks

$$\int \cos(x)\cdot \sin^2(x)\,dx \Rightarrow \int \cos(x)\cdot \left[\sin(x)\right]^2 dx$$

$$\frac{\left[\sin(x)\right]^3}{3} + C$$

$$\frac{1}{3}\left[\sin(x)\right]^3 + C$$

$$\frac{1}{2}\int 2x\,(x^2+1)^5\,dx$$

$$\boxed{\int x^n = \frac{x^{n+1}}{n+1}}$$

Please have a device with you today

Reminders:
- tutoring today after school
- next assessment: Monday

More ex

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \cdot dx \Rightarrow -\int -\sin x \cdot \frac{1}{\cos x} \cdot dx$$

$$-\ln(\cos x) + C$$

$$\ln(\cos(x))^{-1} + C$$

$$\ln|\sec(x)| + C$$

$a \cdot \ln b = \ln b^a$

$\int \frac{1}{x} dx = \ln|x| + C$

Explained in more detail here:

$\frac{1}{\cos} = \sec$

$$\int \tan x \, dx$$

$$\int \frac{\sin x}{\cos x} dx$$

$$\int \sin x \cdot \frac{1}{\cos x} dx \Rightarrow -\int -\sin x \cdot \frac{1}{\cos x} dx \Rightarrow -\ln|\cos x| + C$$

Need $-\sin x$ ...

Log. properties

$$\ln|(\cos x)^{-1}| + C$$

$$\boxed{\ln|\sec x| + C}$$

to Booklet

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \cot x \cdot dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

U-Substitution

$$\int x \cdot \sqrt{2x-1} \cdot dx$$

$$\int x \cdot \underbrace{(2x-1)}_{=}^{1/2} \cdot dx$$

1. Let u = "inside part"
2. Solve for x and solve for dx.
3. Make all replacements

Let $\boxed{u = 2x-1}$

$u+1 = 2x$

$\boxed{\frac{u+1}{2} = x}$

$\frac{d}{dx}(u = 2x-1)$

$\cancel{dx}\left(\frac{du}{\cancel{dx}} = 2\right) dx$

$\frac{du}{2} = \frac{\cancel{2} \cdot dx}{\cancel{2}}$

$\boxed{\frac{du}{2} = dx}$

$\frac{du}{dx} \times \frac{2}{1}$

$2\,dx = du$

$dx = \frac{du}{2}$

$$\int x \cdot (2x-1)^{1/2} \cdot dx$$

$$\int \frac{u+1}{2} \cdot (u)^{1/2} \cdot \frac{du}{2}$$

$$\int \frac{1}{4}(u+1)\, u^{1/2} \cdot du$$

$$\int \frac{1}{4}\left(u^{3/2} + u^{1/2}\right) \cdot du$$

$$\frac{1}{4}\left[\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + C\right]$$

$$\frac{1}{4}\left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C\right]$$

$$\frac{2}{20}u^{5/2} + \frac{2}{12}u^{3/2} + C$$

$$\boxed{\frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + C}$$

Bonus example: using u-sub INSTEAD OF the reverse chain rule

If the reverse chain rule doesn't make sense to you, you can do u-sub instead.

$$\int \sqrt{4x^2-3} \cdot x \, dx$$

$$\int x \cdot (4x^2-3)^{1/2} \, dx$$

Let $u = 4x^2 - 3$

$$\cancel{\frac{u+3}{4} = x^2}$$

$$\cancel{\sqrt{\frac{u+3}{4}} = x}$$

$$\cancel{\frac{\sqrt{u+3}}{2} = x}$$

$$\frac{du}{dx} = 8x$$

$$du = 8x \, dx$$

$$\frac{du}{8x} = dx$$

No need to solve for x here... only when exponents don't work out.

$$\int \boxed{x} (u)^{1/2} \cdot \boxed{\frac{du}{8x}}$$

$$\int \frac{x}{8x} u^{1/2} \cdot du \Rightarrow \int \frac{1}{8} u^{1/2} \, du \Rightarrow \frac{1}{8} \cdot \frac{u^{3/2}}{3/2} + C$$

$$\frac{1}{8} \cdot \frac{2}{3} \cdot u^{3/2} + C$$

$$\frac{1}{12} u^{3/2} + C \Rightarrow \boxed{\frac{1}{12}(4x^2-3)^{3/2} + C}$$

Another u-sub example

$$\int \frac{3x}{4x-2}\,dx$$

$$\int 3x \cdot \frac{1}{4x-2}\,dx$$

Let $\boxed{u = 4x-2}$

$u+2 = 4x$ $\qquad$ $\frac{du}{dx} = 4$

$\boxed{\frac{u+2}{4} = x}$ $\qquad$ $du = 4\,dx$

$\frac{du}{4} = dx$

$\boxed{\frac{1}{4}du = dx}$

$$\int 3 \cdot \overbrace{\frac{u+2}{4}}^{\text{"}x\text{"}} \cdot \frac{1}{u} \cdot \overbrace{\frac{1}{4}du}^{\text{"}dx\text{"}}$$

$$\int \frac{3}{16}(u+2) \cdot u^{-1} \cdot du$$

$$\int \frac{3}{16}\left(1 + 2u^{-1}\right) du$$

$\int \frac{1}{u}\,du = \int u^{-1}\,du = \ln|u| + C$

$$\frac{3}{16}\left[u + 2\ln|u| + C\right]$$

$$\frac{3}{16}u + \frac{6}{16}\ln|u| + C$$

$$\boxed{\frac{3}{16}(4x-2) + \frac{3}{8}\ln\left|4x-2\right| + C}$$

sub example

$$\int \frac{3x+1}{\sqrt{4x+1}}\,dx$$

$$\int (3x+1)\cdot\left(\frac{1}{\sqrt{4x+1}}\right)dx \Rightarrow \int (3x+1)\underline{(4x+1)}^{-1/2}\cdot dx$$

!! ? Let $\boxed{u = 4x+1}$

$u-1 = 4x$

$\boxed{\frac{u-1}{4} = x}$

$\frac{du}{dx} = 4$

$du = 4\,dx$

$\boxed{\frac{1}{4}du = dx}$

$$\int (3x+1)(4x+1)^{-1/2}\cdot dx$$

$$\int \underbrace{\left(3\cdot\frac{u-1}{4}+1\right)}_{\text{Simplify}}(u)^{-1/2}\,\frac{1}{4}du$$

$$\frac{3u-3}{4}+1$$

$$\frac{3}{4}u-\frac{3}{4}+1$$

$$\int \left(\frac{3}{4}u+\frac{1}{4}\right)u^{-1/2}\cdot\frac{1}{4}du$$

Distribute $\frac{1}{4}u^{-1/2}$

$$\int \frac{3}{16}u^{1/2}+\frac{1}{16}u^{-1/2}\cdot du$$

Reverse Power Rule

$$\underbrace{\frac{3}{16}\cdot\frac{2}{3}}\,u^{3/2}+\frac{1}{16}\cdot\frac{2}{1}u^{1/2}+C$$

$$\frac{6}{48}u^{3/2}+\frac{2}{16}u^{1/2}+C$$

$$\frac{1}{8}(4x+1)^{3/2}+\frac{1}{8}(4x+1)^{1/2}+C$$

Replace u's w/ original substitution

$$\boxed{\frac{1}{8}\sqrt{(4x+1)^3}+\frac{1}{8}\sqrt{4x+1}+C}$$

!!