Good afternoon: warm up

$$\int \frac{x^2+4x+4}{x+2}\,dx$$

$$\int \frac{(x+2)\cancel{(x+2)}}{\cancel{x+2}}\,dx$$

$$\int (x+2)\,dx$$

$$\boxed{\frac{1}{2}x^2+2x+C}$$

$$\int (x^2+6x+9)^5\,dx$$

$$\int \left((x+3)(x+3)\right)^5\,dx$$

$$(x+3)^2$$

$$\int \left((x+3)^2\right)^5\,dx$$

$$\int (x+3)^{10}\cdot dx$$

want: 1

$$\int x^n = \frac{x^{n+1}}{n+1}+C$$

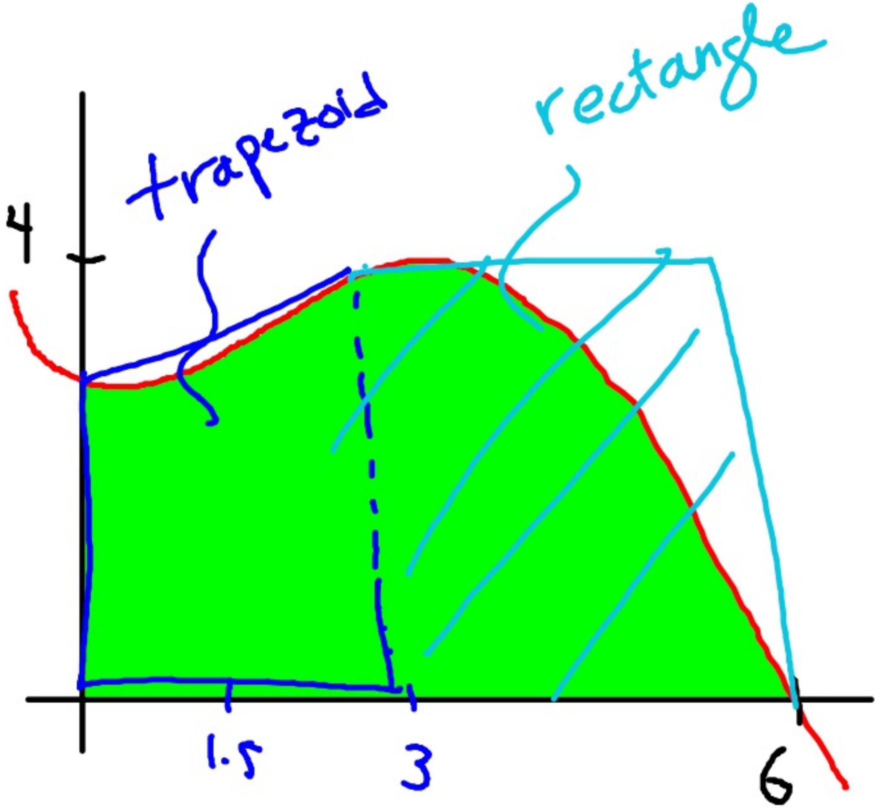
$$\boxed{\frac{1}{11}(x+3)^{11}+C}$$

Finding C hw answers

53. time is 1.875 sec, height is 62.25 ft
54. initial velocity is 187.617 ft/s (this one's hard, sorry)
55. a.) 2.562 sec (must use quad. formula)
    b.) -65.970 ft/s

56. 7.1m
57. 62.3 m/sec (also hard, sorry)
58. 9.2 sec

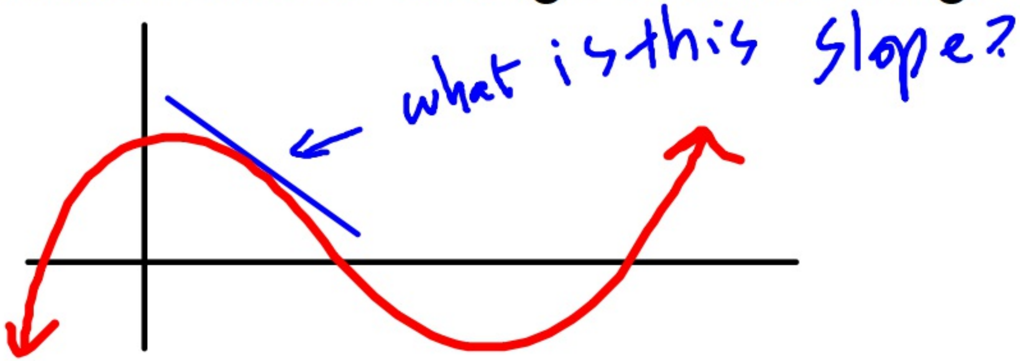
Sketch the figure then estimate the green area.



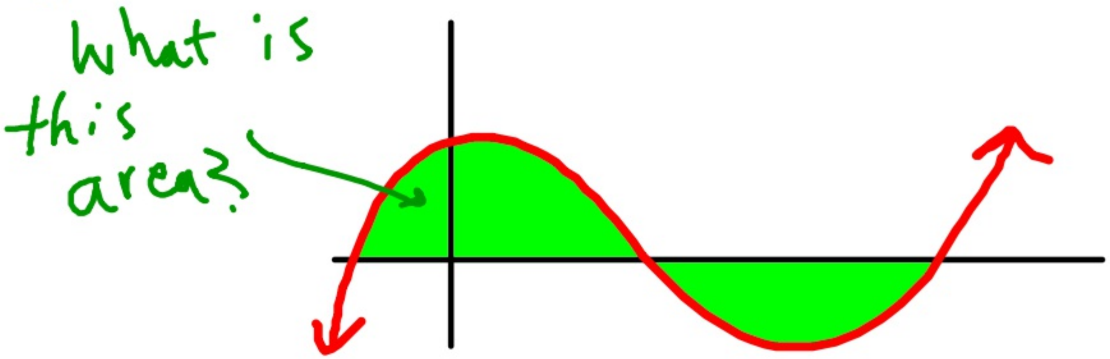
divide region into
smaller shapes with geometric
area formulas and find those
areas, add them together

[redacted]

Differential Calculus: rates of change and the tangent line problem



Integral Calculus: accumulation and the area problem



Remember limits?

limits help make static things dynamic!!

contrast f(0) vs $\lim_{x \to 0} f(x)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_at_a_point.html

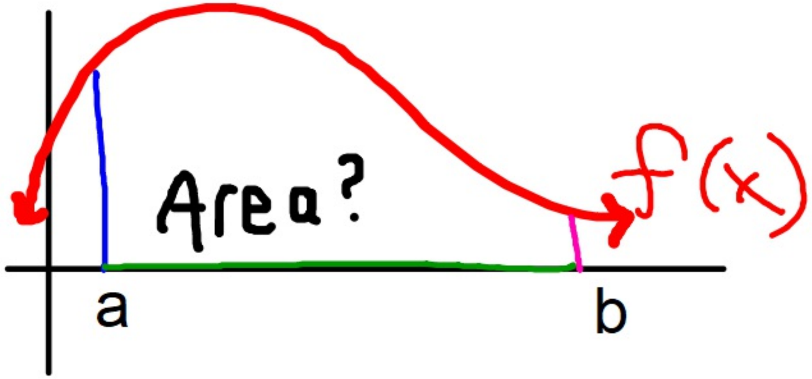
limit def of derivative

integration/area
also involves limits

http://webspace.ship.edu/msrenault/GeoGebraCalculus/integration_riemann_sum.html

Estimating Area using Riemann Sums

Estimate the area of the region bounded by f(x), x=a, x=b, and the x-axis.



1826-66

https://en.wikipedia.org/wiki/List_of_things_named_after_Bernhard_Riemann

#tbt

Summation Notation summ-ary 😎

Write as a sum using sigma notation.

$$1+2+3+4+5+6$$

stop

$$\sum_{i=1}^{6} i$$

start

index

Write as a sum using sigma notation:

$5+10+15+20+25+30 \Rightarrow 5(1)+5(2)+5(3)+\cdots$

$5(\underbrace{1+2+3+4+5+6})$

$= 5\sum_{i=1}^{6} i \Leftarrow$

$\sum_{i=1}^{6} 5i$

$\sum_{i=0}^{5} 5(i+1)$

$5 + 10 + 15 + 20 + 25 + 30 + \ldots + 5n = \sum_{i=1}^{n} 5i$

1 2 3 4 n

* summation formulas

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

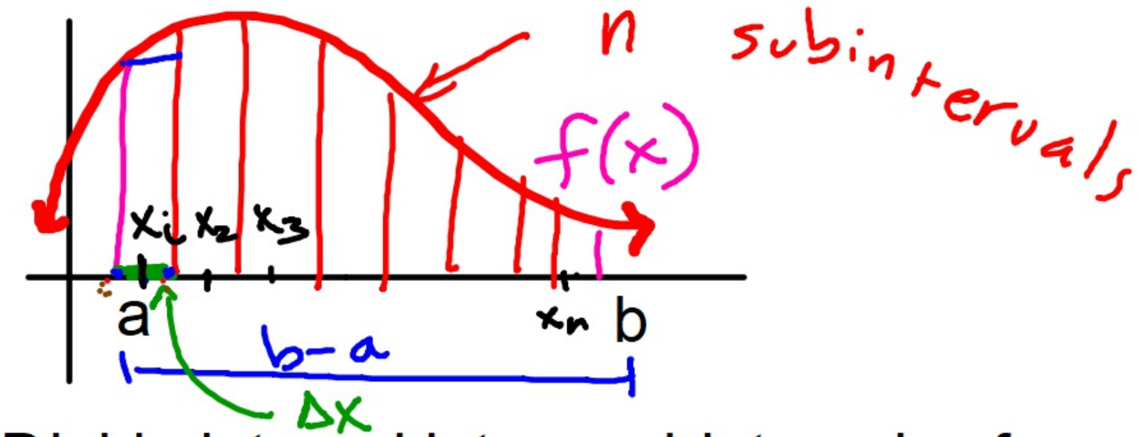
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Write as a sum using sigma notation:

5 + 10 + 15 + 20 + 25 + 30 + ...

$$\sum_{i=1}^{\infty} 5i \longrightarrow \lim_{n\to\infty} \sum_{i=1}^{n} 5i$$



Divide interval into *n* subintervals of equal size (equality not necessary)
this represents base of rectangle

$$\Delta x = \frac{b-a}{n}$$

Use function to generate height of rectangle at specific points $f(x_i)$

https://www.desmos.com/calculator/t4cysp2e1n

Area of one rectangle: height * base = $f(x_i)$ *Δx

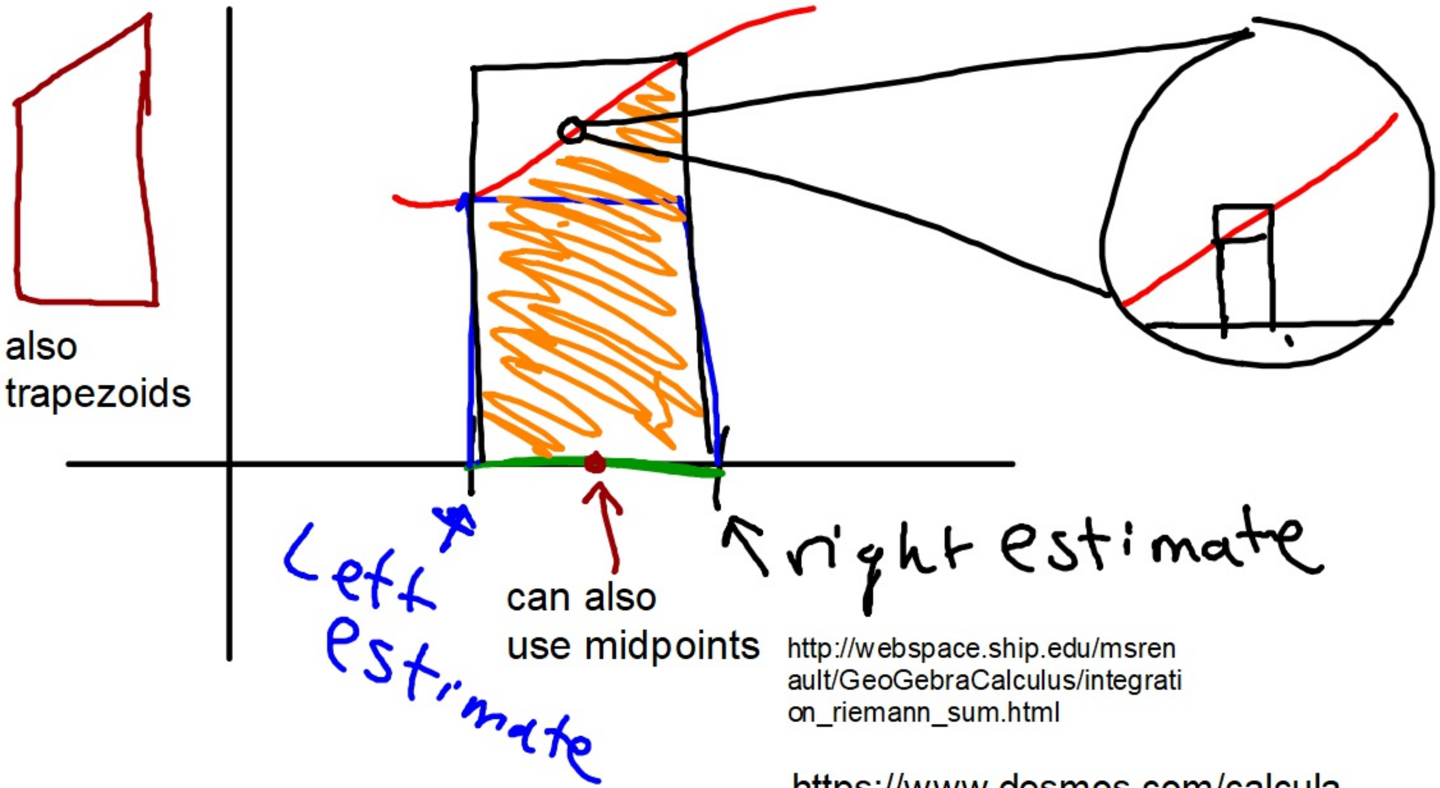
Area of one rectangle: $f(x_i) * \Delta x$

Area of n rectangles:

$$\sum_{i=1}^{n} \boxed{f(x_i)\Delta x} \quad where \;\; \Delta x = \frac{b-a}{n}$$

The area under f(x) between a and b can be approximated with *n* rectangles by this

How to choose the height?



can also
use trapezoids

can also
use midpoints

http://webspace.ship.edu/msrenault/GeoGebraCalculus/integration_riemann_sum.html

https://www.desmos.com/calculator/ercx7noboq

$$\sum_{i=1}^{n} f(x_i)\Delta x \quad where\ \Delta x = \frac{b-a}{n}$$

How do we move from area approximation to exact area?

make $\underline{n} \to \infty$

$$\Delta x = \frac{b-a}{n}$$

"Subintervals"
"# of rectangles"

Riemann Definition of Definite Integral

width of interval

$$\Delta x = \frac{b-a}{n} \quad ; \quad \underbrace{\lim_{n\to\infty}}_{} \sum_{i=1}^{n} \boxed{f(x_i)\Delta x} = \int_a^b \boxed{f(x)\,dx}$$

∞ rectangles

value in subint.

Exact Area under curve

width of subinterval

# of subintervals

height of 1 rectangle

base of 1 rectangle

Infinite sum

Area of 1 rectangle

Area of n rectangles

A future assessment question:

I-U1

3. The Riemann definition of the definite integral is given by

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \underbrace{\int_a^b f(x)dx}_{}$$

where $\Delta x = \frac{b-a}{n}$. Explain why this is so in the context of area under a curve. You may use diagrams to accompany your explanation.

called a "definite integral"

Indefinite Integrals vs Definite Integrals

ex

tbd

Type
of answer:

We have not yet learned how to do math with $\int_a^b f(x)\,dx$

We use Riemann sums to approximate the definite integral

How to approximate area under a curve

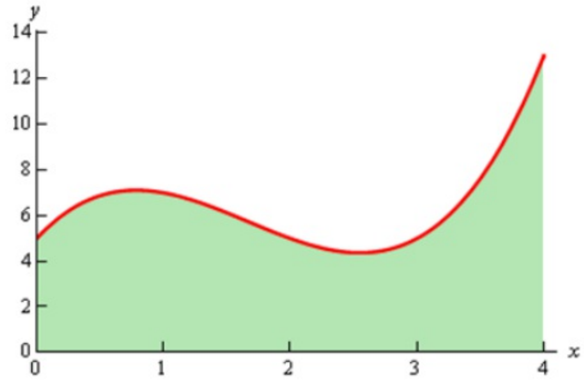
Estimate the area under the curve $f(x) = x^3 - 5x^2 + 6x + 5$
on the interval [0,4] using 4 subintervals of equal width.

a b n

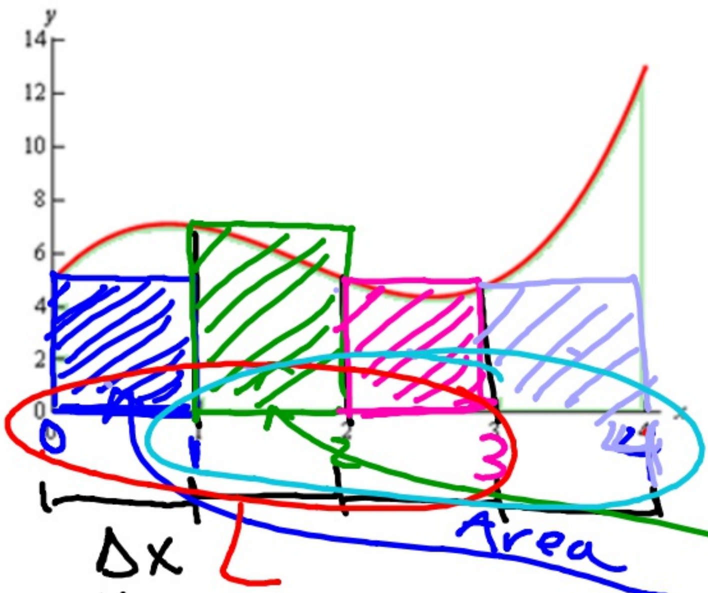
Write approximation as a Riemann sum

Write as a Definite Integral

$$\underbrace{\sum_{i=1}^{4} f(x_i)\cdot\Delta x}_{\text{estimate}} \approx \underbrace{\int_0^4 f(x)\,dx}_{\text{exact area}}$$



Left-Rectangle Approximation Method (LRAM) : $f(x) = x^3 - 5x^2 + 6x + 5$



$$\Delta x = \frac{b-a}{n} = \frac{4-0}{4} = 1$$

LRAM:
consider each subinterval's endpoint,
choose the LEFT (or lesser) value
0 to 1? choose 0
1 to 2? choose 1
etc.

Area

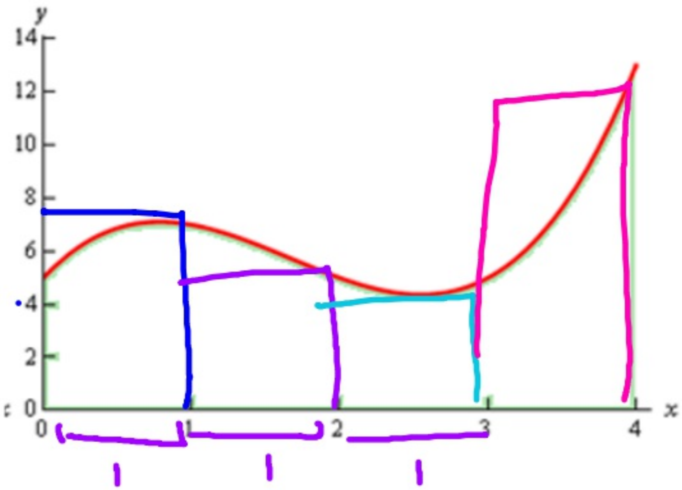
$$\sum_{i=1}^{4} f(x_i)\Delta x = f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1$$

$$5 + 7 + 5 + 5$$

$$= 22 \leftarrow \text{LRAM estimate of } \int_0^4 f(x)\,dx$$

Right-Rectangle Approximation Method (RRAM)

$f(x) = x^3 - 5x^2 + 6x + 5$



RRAM:
consider each subinterval's endpoint,
choose the RIGHT (or bigger) value
0 to 1? choose 1
1 to 2? choose 2
etc.

$f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1$

$7 + 5 + 5 + 13 = 30$ ← RRAM estimate for $\int_0^4 f(x)\,dx$

Approximate $\int_2^4 \ln x \, dx$ using 4 left rectangles

$\Delta x = \frac{4-2}{4} = \frac{2}{4} = \frac{1}{2}$ ← interval size

2 – 2.5 – 3 – 3.5 – 4

choose the left of each interval
to be the input for f(x)
remember to multiply each by delta x

$f(2) \cdot \frac{1}{2} + f(2.5) \cdot \frac{1}{2} + f(3) \cdot \frac{1}{2} + f(3.5) \cdot \frac{1}{2}$

= answer

HW
p. 263 #25-30, 33-34

Do LRAM and RRAM for each