Good afternoon: warm up

Subinterval width

width of interval

Area of n rectangles

rect. base

exact area of space

$$\Delta x = \frac{b-a}{n}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x = \int_a^b f(x)\,dx$$

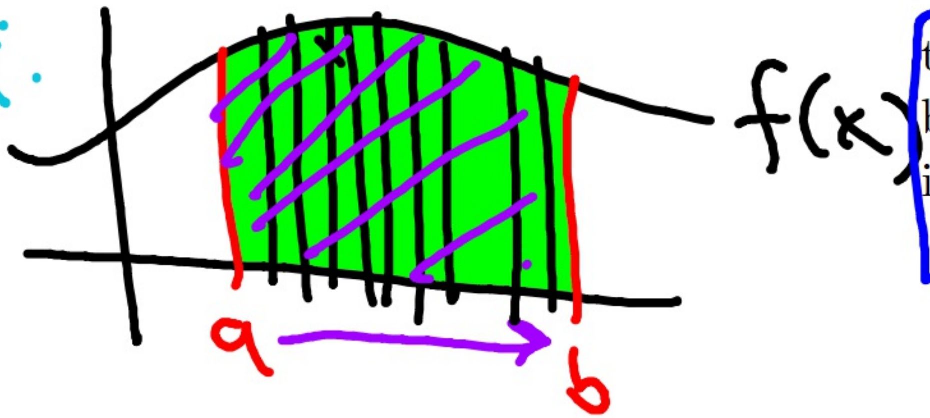
\# of subints

rect. height

1 rect. Area

inf. rect.

Explain the meaning of the Riemann definition of definite integral in the context of the exact area under the curve f(x) on the interval [a,b] with n subintervals.



the area under a curve can be found by adding the areas of infinitely many, inf. thin rectangles.

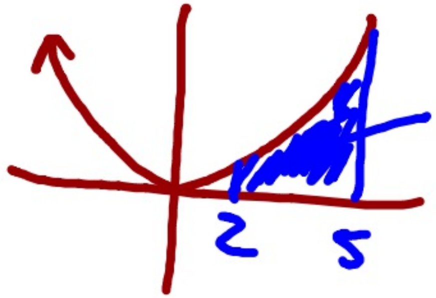
Indefinite Integrals vs Definite Integrals

ex $\int x^2 \cdot dx$

$= \frac{1}{3}x^3 + C$

Type of answer: Family of functions

$\int_2^5 x^2 \cdot dx$

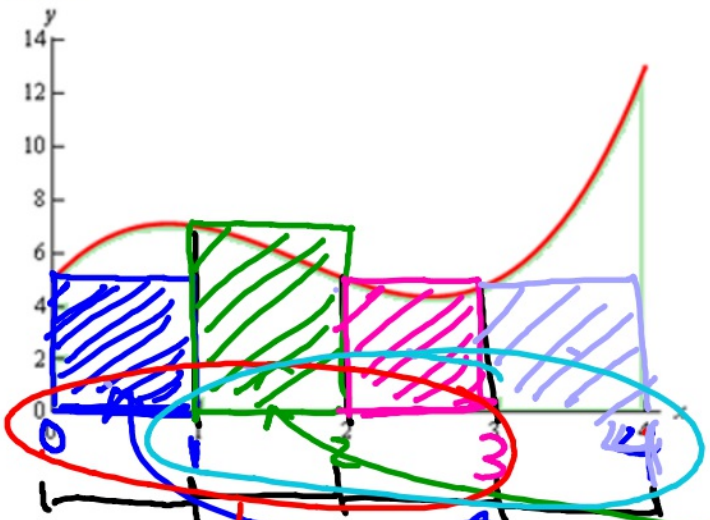
$=$ 

?

$=$ Number answer.

Last time.......

Left-Rectangle Approximation Method (LRAM) : $f(x) = x^3 - 5x^2 + 6x + 5$



$$\Delta x = \frac{b-a}{n} = \frac{4-0}{4} = 1$$

LRAM:
consider each subinterval's endpoint,
choose the LEFT (or lesser) value
0 to 1? choose 0
1 to 2? choose 1
etc.

Δx

Area

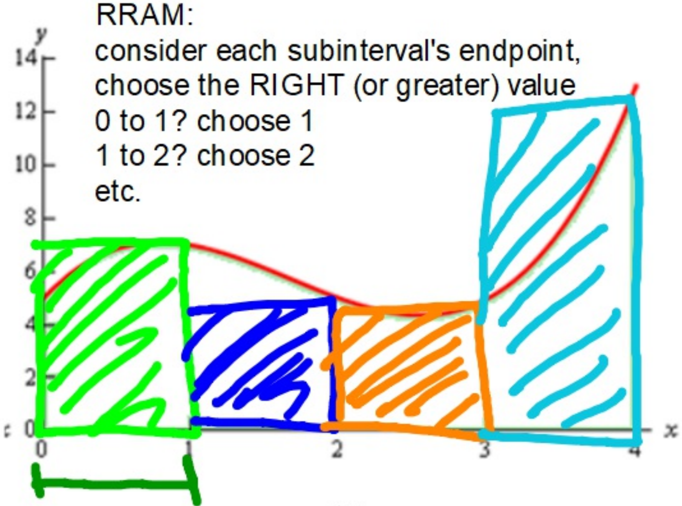
Area

$$\sum_{i=1}^{4} f(x_i)\Delta x = f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1$$

$$5 + 7 + 5 + 5$$

$= 22$ ← LRAM estimate of $\int_0^4 f(x)\,dx$

Right-Rectangle Approximation Method (RRAM)



$f(x) = x^3 - 5x^2 + 6x + 5$

$[0,4] \quad n = 4. \quad \Delta x = \frac{4-0}{4} = 1$

$$\int_0^4 x^3 - 5x^2 + 6x + 5 \cdot dx \approx$$

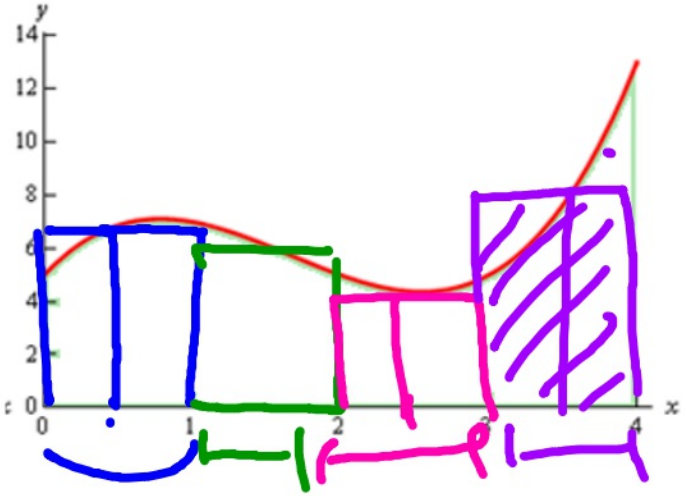
$$\approx \sum_{i=1}^{4} f(x_i)\cdot \Delta x = f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1$$

$$\approx 7 + 5 + 5 + 13$$

$$\boxed{\approx 30}$$

~~Right~~ Mid Rectangle Approximation Method (~~RRAM~~ MRAM)

$f(x) = x^3 - 5x^2 + 6x + 5$



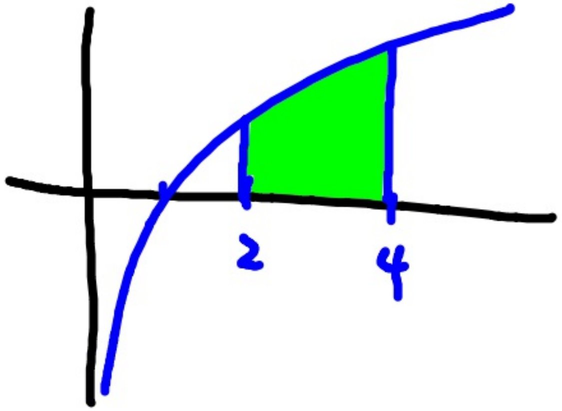
MRAM:
consider each subinterval's endpoint,
choose the midpoint for the input
0 to 1? choose 0.5
1 to 2? choose 1.5
etc.

$$f(0.5)\cdot 1 + f(1.5)\cdot 1 + f(2.5)\cdot 1 + f(3.5)\cdot 1$$

$\approx$ 

Approximate $\int_{2}^{4} \ln x \, dx$ using 4 right rectangles

$b = 4$, $a = 2$, $n = 4$



$$\Delta x = \frac{4-2}{4} = 0.5$$

$$2 - 2.5 - 3 - 3.5 - 4$$

$$\approx \ln(2.5)\cdot 0.5 + \ln(3)\cdot 0.5 + \ln(3.5)\,0.5 + \ln(4)\cdot 0.5$$

$\approx$  $\rightarrow$

Approximate $\int_2^4 \ln x \, dx$ using 4 MIDPOINT rectangles

Understand calculus concepts...

...verbally

...numerically

...algebraically

...graphically

Be fluent algebraically:

Write a definite integral whose value equals

$$\lim_{n\to\infty} \sum_{i=1}^{n} \Big[\big(3+\tfrac{5}{n}i\big)^3 - \big(3+\tfrac{5}{n}i\big)^2 - 4\Big]\frac{5}{n}$$

a

VARIABLE VALUES AS $i \to n$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\boxed{\Delta x} = \int_a^b f(x)\,dx$$

$$\Delta x = \frac{b-a}{n} = \frac{5}{n}$$

$a = 3 \longrightarrow b = 8$

$$\int_3^8 \underbrace{x^3 - x^2 - 4}\,dx$$

Write a definite integral whose value equals

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(6+\frac{2i}{n}\right)\frac{2}{n}$$

$$\int_6^8 \sin(x)\,dx$$

Numerical Fluency:

Suppose f(x) is a differentiable function. Here are some values:

x	-1	2	3	5	7	10
f(x)	9	10	2	-1	-3	3

Estimate $\int_{-1}^{10} f(x)\,dx$ using a right Riemann sum.

HW
p263 #25-30, 33,34
do an LRAM and RRAM for each