

Good afternoon: warm up

Subinterval width $\Delta x = \frac{b-a}{n}$

width of interval $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_a^b f(x) dx$

Area of rect. base $f(x_i) \Delta x$

Area of n rectangles $\sum_{i=1}^n f(x_i) \Delta x$

exact area of space $\int_a^b f(x) dx$

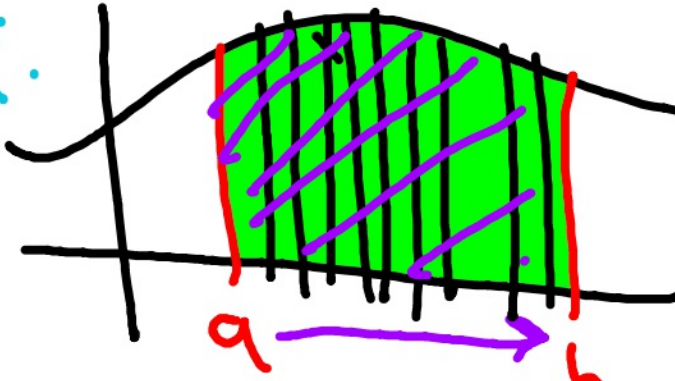
of subints n

height $f(x_i)$

1 rect. Area $f(x_i) \Delta x$

Explain the meaning of the Riemann definition of definite integral in the context of the exact area under the curve $f(x)$ on the interval $[a,b]$ with n subintervals.

inf. rect.



$f(x)$

the area under a curve can be found by adding the areas of infinitely many, inf. thin rectangles.

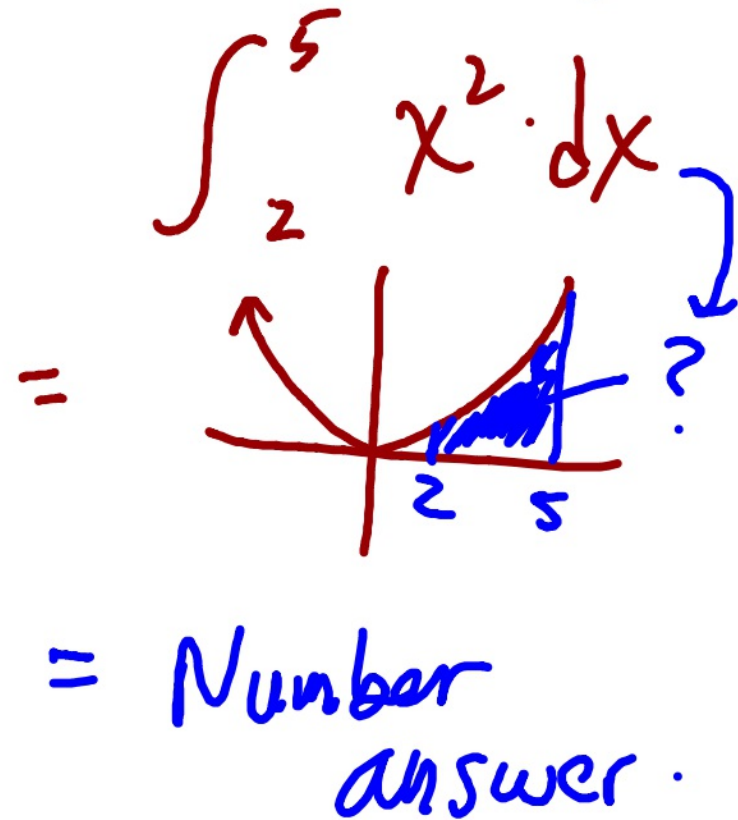
Indefinite Integrals

vs

Definite Integrals

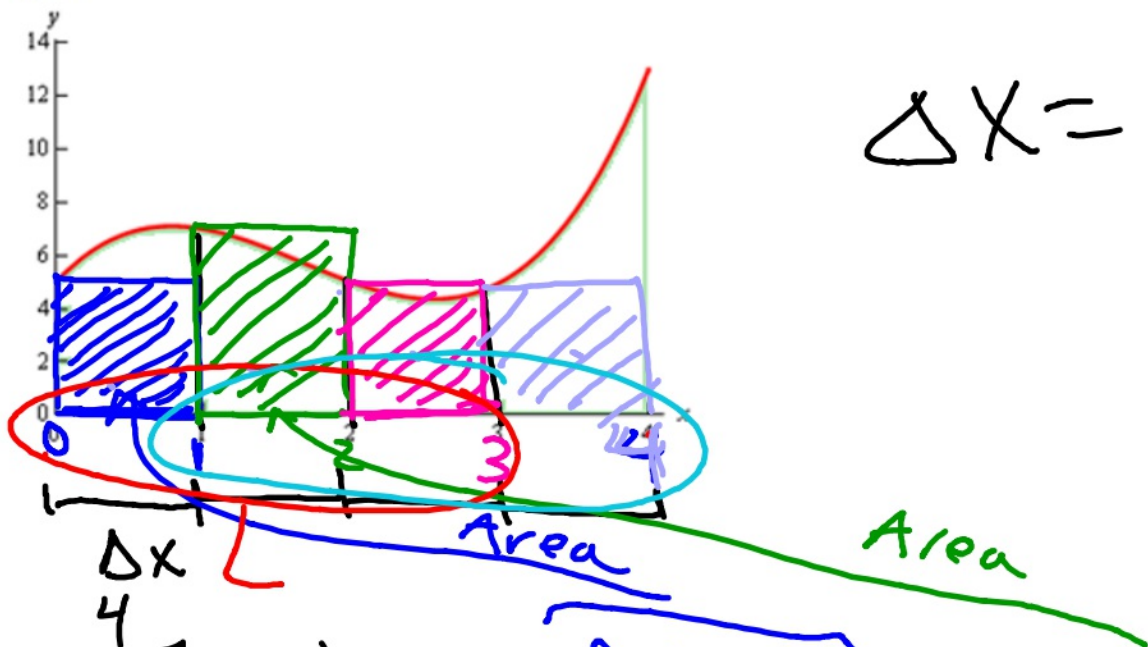
ex $\int x^2 \cdot dx$
 $= \frac{1}{3}x^3 + C$

Type of answer: Family of functions



Last time.....

Left-Rectangle Approximation Method (LRAM) : $f(x) = x^3 - 5x^2 + 6x + 5$



$$\Delta x = \frac{b-a}{n} = \frac{4-0}{4} = 1$$

LRAM:

consider each subinterval's endpoint,
choose the LEFT (or lesser) value

0 to 1? choose 0

1 to 2? choose 1

etc.

$$\sum_{i=1}^4 f(x_i) \Delta x = f(0) \cdot 1 + f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1$$

$$= 5 + 7 + 5 + 5 \leftarrow \text{LRAM estimate of } \int_0^4 f(x) dx$$

(Note: The value 22 is circled in the original image.)

Right-Rectangle Approximation Method (RRAM)

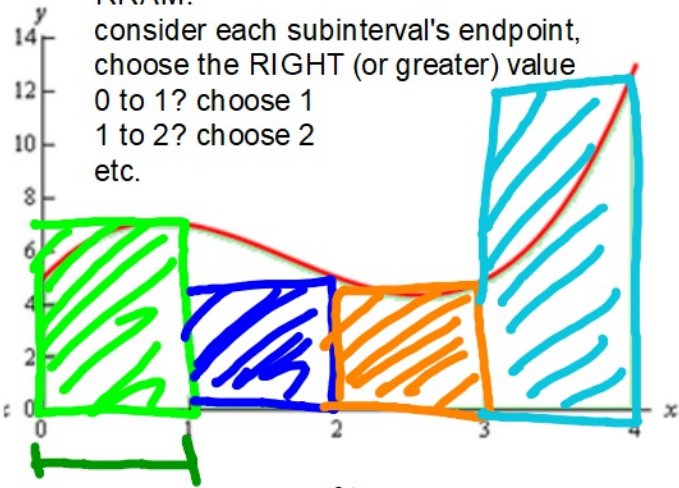
RRAM:

consider each subinterval's endpoint,
choose the RIGHT (or greater) value

0 to 1? choose 1

1 to 2? choose 2

etc.



$$f(x) = x^3 - 5x^2 + 6x + 5$$

$$[0, 4]$$

$$n = 4$$

$$\Delta x = \frac{4-0}{4} = 1$$

$$\int_0^4 x^3 - 5x^2 + 6x + 5 \cdot dx \approx$$

$$\approx \sum_{i=1}^4 f(x_i) \cdot \Delta x = f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1$$

$$\approx 7 + 5 + 5 + 13$$

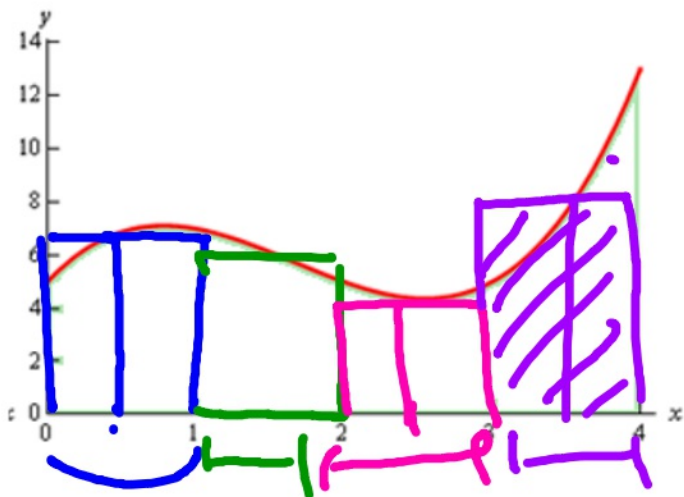
$\boxed{= 30}$

Mid

MRAM

~~Rect~~ Rectangle Approximation Method (~~R~~RAM)

$$f(x) = x^3 - 5x^2 + 6x + 5$$



MRAM:

consider each subinterval's endpoint,
choose the midpoint for the input

0 to 1? choose 0.5

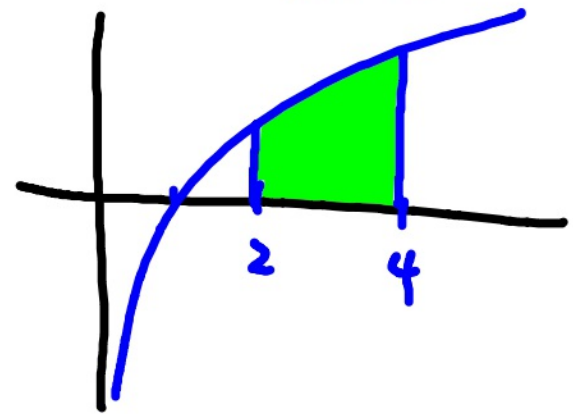
1 to 2? choose 1.5

etc.

$$f(0.5) \cdot 1 + f(1.5) \cdot 1 + f(2.5) \cdot 1 + f(3.5) \cdot 1$$



Approximate $\int_a^b \ln x \, dx$ using 4 right rectangles



$$\Delta x = \frac{4-2}{4} = 0.5$$

2 - 2.5 - 3 - 3.5 - 4

$$\approx \ln(2.5) \cdot 0.5 + \ln(3) \cdot 0.5 + \ln(3.5) \cdot 0.5 + \ln(4) \cdot 0.5$$

\approx  \rightarrow

Approximate $\int_2^4 \ln x \, dx$

using 4 MIDPOINT rectangles

Understand calculus concepts...

✓...verbally

...numerically

—...algebraically

✓...graphically



Be fluent algebraically:

Write a definite integral whose value equals

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(3 + \frac{5}{n}i\right)^3 - \left(3 + \frac{5}{n}i\right)^2 - 4 \right] \frac{5}{n}$$

Handwritten notes: "a" with an arrow pointing to the 3 in the first term; "VARIABLE VALUES AS $i \rightarrow n$ " written above the sum; blue circles around the terms in the sum; a red circle around the $\frac{5}{n}$ term.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_a^b f(x) dx$$

Handwritten notes: The expression is enclosed in a cloud-like border. A red box highlights Δx in the sum.

$$\Delta x = \frac{b-a}{n} = \frac{5}{n}$$

$$a = 3 \rightarrow b = 8$$

$$\int_3^8 (x^3 - x^2 - 4) dx$$

Handwritten notes: The integral is circled in blue. A horizontal line is drawn under the integrand $x^3 - x^2 - 4$.

Write a definite integral whose value equals

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin\left(6 + \frac{2i}{n}\right) \frac{2}{n}$$

$$\int_6^8 \sin(x) dx$$

Numerical Fluency:

Suppose $f(x)$ is a differentiable function. Here are some values:

x	-1	2	3	5	7	10
$f(x)$	9	10	2	-1	-3	3

Estimate $\int_{-1}^{10} f(x) dx$ using a right Riemann sum.

HW

p263 #25-30, 33,34

do an LRAM and RRAM for each