Good afternoon: please do warm up in notebooks

$$\int (x^2 + 14x + 49)^{35} dx$$

$$\int \left((x+7)^2 \right)^{35} dx$$

$$\int 1 \cdot (x+7)^{70} \, dx$$

$$\frac{(x+7)^{71}}{71} + C$$

U-Sub (notes)

$$\int x^2 \sqrt{1-x} \quad dx$$

$$\int x^2 (1-x)^{1/2} dx$$

???

Let $u = 1-x$

$x = 1-u$

$\left(\frac{du}{dx} = -1\right) dx$

$du = -dx$

$-du = dx$

$$\int (1-u)^2 u^{1/2} \cdot -du$$

$$-\int \boxed{(1-u)^2} u^{1/2} \cdot du$$

$$-\int (1-2u^1+u^2) u^{1/2} \cdot du$$

$$-\int u^{1/2} - 2u^{3/2} + u^{5/2} du$$

$$-\left[\frac{2}{3}u^{3/2} - 2 \cdot \frac{2}{5}u^{5/2} + \frac{2}{7}u^{7/2} + C\right]$$

$$\boxed{-\frac{2}{3}u^{3/2} + \frac{4}{5}u^{5/2} - \frac{2}{7}u^{7/2} + C}$$

Ax^n


And now for something completely different

Summation Notation summ-ary

Write as a sum using sigma notation.

$$1+2+3+4+5+6$$

where to end

$$\sum_{i=1}^{6} i$$

what to do w/ each number

where to start

Write as a sum using sigma notation:

5+10+15+20+25+30

5(1+2+3+4+5+6)

[redacted]

$$\sum_{i=1}^{6} 5i = 5\sum_{i=1}^{6} i$$

5(1)+5(2)+5(3)+5(4)+5(5)+5(6)

[redacted]

Write as a sum using sigma notation:

$5 + 10 + 15 + 20 + 25 + 30 + \ldots + 5n$

$$= \sum_{i=1}^{n} 5i$$

Write as a sum using sigma notation:

5 + 10 + 15 + 20 + 25 + 30 + ...

$$\sum_{i=1}^{\infty} 5i \Rightarrow \lim_{n \to \infty} \sum_{i=1}^{n} 5i$$

Indefinite Integration vs Definite Integration

$$\int x+3 \, dx$$

$$\int_1^5 x+3 \, dx$$

Answer:

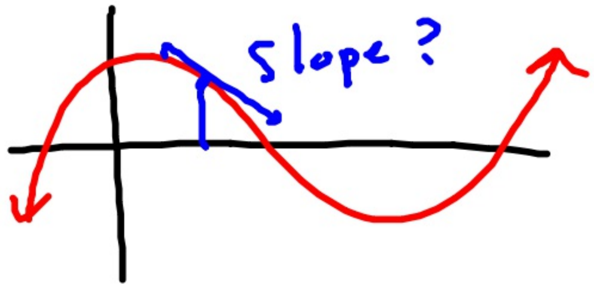
family of functions

$$\underline{\frac{1}{2}x^2+3x+C}$$

Answer

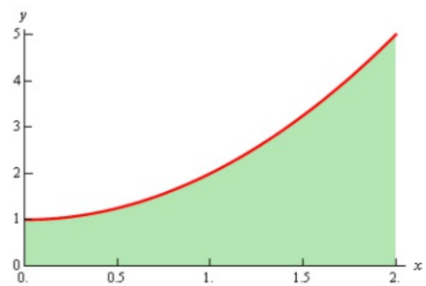
Number

Derivatives: rates of change and the tangent line problem

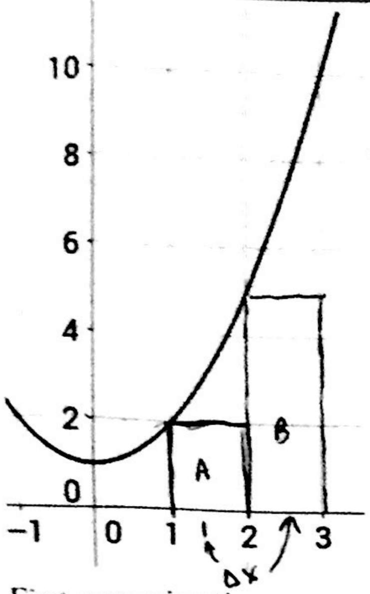


Definite integrals: summation and the area problem

accumulation



How to find the area under a curve?



First approximation

Find a method to approximate the area enclosed by $y = x^2 + 1$, the vertical lines x=1 and x=3, and the x-axis.
(This is called 'the area under the curve')

rect A   rect B

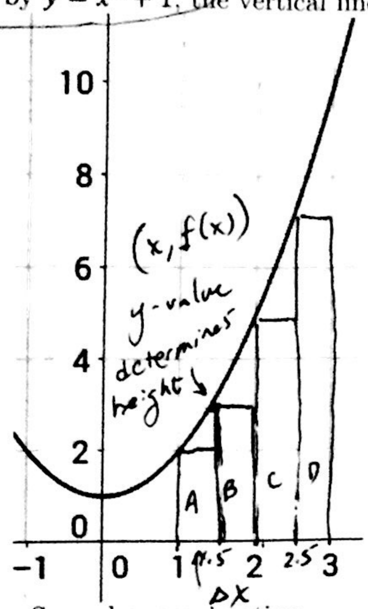
$$\text{Area} \approx \underbrace{1\cdot(2)}_{\text{rect } A} + \underbrace{1\cdot(5)}_{\text{rect } B}$$

base   f(1), or height   base   f(2) or height

$$\approx 2 + 5$$

$$\approx 7 \text{ sq. units}$$

How can you improve your method?
more, smaller rectangles.



Second approximation

$$\text{Area} \approx \underbrace{\tfrac{1}{2}(f(1))}_{\text{Rect. } A} + \underbrace{\tfrac{1}{2}(f(1.5))}_{B} + \underbrace{\tfrac{1}{2}(f(2))}_{C} + \underbrace{\tfrac{1}{2}\cdot(f(2.5))}_{D}$$

Base height

$$\approx \frac{1}{2}\cdot 2 + \frac{1}{2}(3.25) + \frac{1}{2}(5) + \frac{1}{2}(7.25)$$

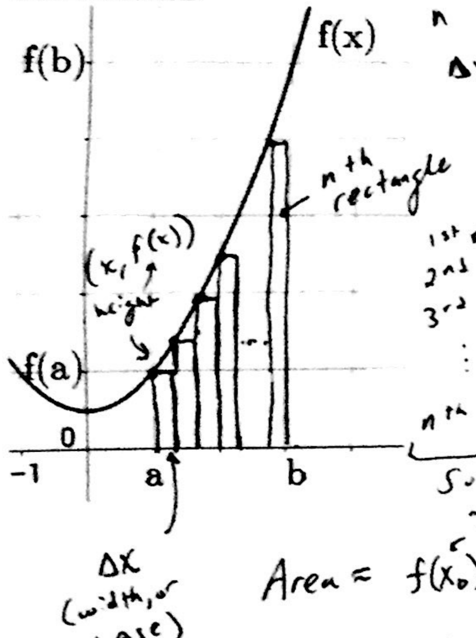
$$\approx \frac{1}{2}(2 + 3.25 + 5 + 7.25)$$

$$\approx \frac{1}{2}(17.5)$$

$$\approx 8.75 \text{ sq. units}$$

think about $\sum_{i=1}^{4}$

Generalization:



n rectangles; split [a, b]

$$\Delta x = \frac{b-a}{n}$$

Area

1st rectangle: $\Delta x \cdot f(a)$

2nd rectangle: $\Delta x \cdot f(a + \Delta x)$

3rd rectangle: $\Delta x \cdot f(a + 2\Delta x)$

⋮

nth rectangle $\Delta x \cdot f(b - \Delta x)$

Sum total

$$\text{Area} \approx f(x_0)\Delta x + f(x_1)\Delta x + \dots + f(x_n)\Delta x$$

"a"   "b"

$$= \Delta x \left(f(x_0) + f(x_1) + \dots + f(x_n)\right)$$

$$= \Delta x \cdot \sum_{i=1}^{n} f(x_i) \underset{\text{commutative property}}{=} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

$$\int_a^b f(x)\cdot dx$$

"S" $\sum_{i=1}^{n} f(x_i)\cdot\Delta x$

Riemann definition of Definite Integral: if f is a continuous function on [a,b] and this interval is equally divided into n intervals of width $\Delta x = \frac{b-a}{n}$, and if $x_i = a + i\Delta x$ is the right endpoint of subinterval i, then:

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$$

← Infinite Rectangles!