

SOL5

I-A3

1. Suppose $f'(x) = 2\sqrt{x}$ and $f(1) = 4$. Find the value of $f(4)$.

$$\int f'(x) dx = f(x) + C$$

$$\int 2\sqrt{x} dx =$$

$$\int 2x^{1/2} dx$$

$$2 \int x^{1/2} dx \quad) \text{ rev. power rule}$$

$$2 \left[\frac{x^{3/2}}{3/2} + C \right]$$

$$2 \cdot \frac{2}{3} x^{3/2} + C \rightarrow f(x) = \frac{4}{3} x^{3/2} + C$$

use $(1, 4)$ to find C
 $f(1) = 4 = \frac{4}{3}(1)^{3/2} + C$

$$4 = \frac{4}{3} + C$$

$$\frac{8}{3} = C$$

$$f(x) = \frac{4}{3} x^{3/2} + \frac{8}{3}$$

$$f(4) ?$$

$$= \frac{4}{3}(4)^{3/2} + \frac{8}{3}$$

$$\frac{32}{3} + \frac{8}{3}$$

$$= \frac{40}{3}$$

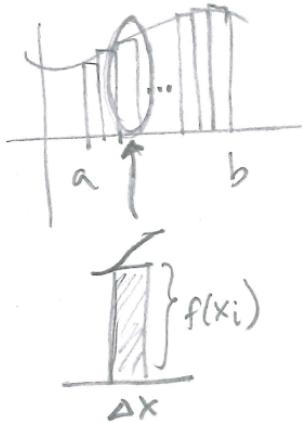
I-U1

2.

The Riemann definition of the definite integral is given by

$$\lim_{n \rightarrow \infty} \sum_{i=0}^n f(x_i) \Delta x = \int_a^b f(x) dx$$

where $\Delta x = \frac{b-a}{n}$. Explain why this is so in the context of area under a curve. You may use diagrams to accompany your explanation.



Interval size is $b-a$. Split into n subintervals, called Δx . Rectangles have Δx as base and $f(x_i)$ as height, giving $f(x_i)\Delta x$ as area. $\sum_{i=1}^n$ gives sum of n -rectangle's area. To give exact area, increase $n \rightarrow \infty$ so each rectangle is thin, and there are ∞ many of them. This gives the exact area, $\int_a^b f(x) dx$.

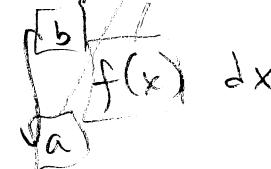
I-U2

3. Write a definite integral whose value equals

$$\lim_{n \rightarrow \infty} \sum_{i=0}^n \left[2\left(5 + \frac{4i}{n}\right)^2 + 3\left(5 + \frac{4i}{n}\right) \right] \frac{4}{n} \quad b-a$$

$$\Delta x = \frac{b-a}{n}$$

Need these



$$a=5 \rightarrow b-a=4, \text{ so } b=9$$

$$2(x)^2 + 3(x)$$

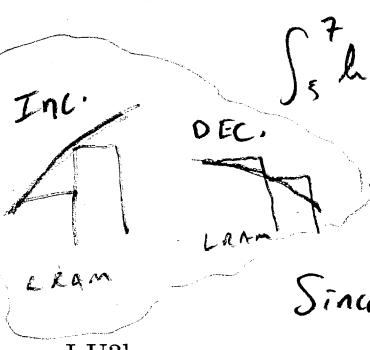
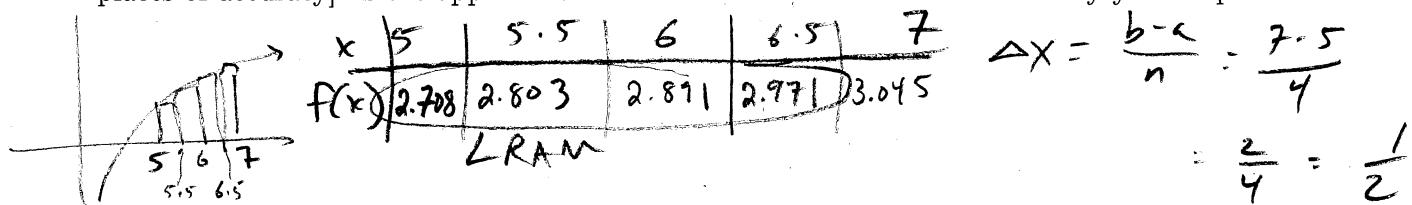
$$f(x)$$

$$\int_5^9 2x^2 + 3x \, dx$$

I-U3a

LRAM

4. Find the left rectangle approximation for $\int_5^7 \ln(3x) dx$ using 4 rectangles of equal width. [3 decimal places of accuracy]. Is the approximation an under or an overestimate? Justify your response.



Since $f(x)$ is increasing, LRAM is an underapproximation.

I-U3b

5. Find the midpoint rectangle approximation for $\int_3^7 \tan(0.2x) dx$ using 4 rectangles of equal width. [3 decimal places of accuracy].

$$\Delta x = \frac{7-3}{4} = \frac{4}{4} = 1$$

x	3	3.5	4	4.5	5	5.5	6	6.5	7
$f(x)$	1.684	1.842	1.030	1.260	1.557	1.765	2.572	3.602	5.780

For any given

$$\int_3^7 \tan(0.2x) dx \approx \Delta x \sum f(x)$$

$$\approx 1 \cdot [1.842 + 1.260 + 1.765 + 3.602]$$

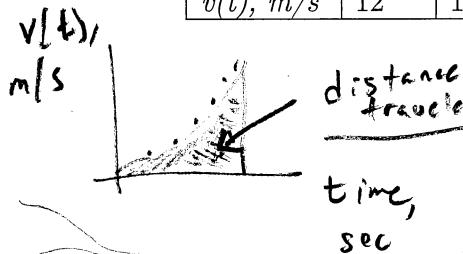
$$\boxed{7.669}$$

Subinterval, use the
midpoint of the
subinterval to determine
the y-values.

I-U3c

6. An awesome rocket ship is in the air and doing cool rocket things. Its velocity $v(t)$ is a differentiable, strictly increasing function. Selected values are given below. Using correct units, explain the meaning of $\int_2^{10} v(t) dt$ in the context of this problem. Then, approximate the value of $\int_2^{10} v(t) dt$ using the 4 trapezoids indicated by the table.

t , sec	2	4	6	8	10
$v(t)$, m/s	12	18	27	38	52



$\int_2^{10} v(t) dt$ is the distance
the rocket travels, in meters,
from 2 sec to 10 sec.

$$\Delta x = \frac{10-2}{4} = \frac{8}{4} = 2$$

$$\int_2^{10} v(t) dt \approx \frac{2}{2} [12 + 2(18 + 27 + 38) + 52]$$

$$\approx 1 [12 + 2(83) + 52]$$

$$= 230 \text{ meters}$$

TRAP

$$\frac{\Delta x}{2} [f(a) + 2(f(x_1) + \dots) + f(b)]$$

I-U5

Evaluate each definite integral. Show all work and simplify your answer.

$$7. \int_4^9 2x - \frac{1}{\sqrt{x}} dx = \int_4^9 2x - \frac{1}{x^{1/2}} dx \rightarrow \int_4^9 2x - x^{-1/2} dx$$

$$\left[x^2 - 2x^{1/2} \right]_4^9$$

$$\left[9^2 - 2(9)^{1/2} \right] - \left[4^2 - 2(4)^{1/2} \right]$$

$$f(9) \qquad \qquad \qquad f(4)$$

$$8. \int_2^5 \frac{3x^2 - 4x}{x} dx$$

$$81 - 2(3) - (16 - 4)$$

$$\int_2^5 \frac{3x^2}{x} - \frac{4x}{x} dx$$

$$75 - 12 = \textcircled{63}$$

$$\int_2^5 3x - 4 dx \Rightarrow \left[\frac{3}{2}x^2 - 4x \right]_2^5 \Rightarrow \left[\frac{3}{2}(5^2) - 4(5) \right] - \left[\frac{3}{2}(2^2) - 4(2) \right]$$

$$17.5 - -2 = \textcircled{19.5}$$

I-U7

Suppose $f(x)$ and $h(x)$ are continuous functions such that

$$\int_1^9 f(x) dx = -1, \quad \int_7^9 f(x) dx = 5, \quad \int_7^9 h(x) dx = 4.$$

$$9. \int_9^7 [h(x) - f(x)] dx$$

$$= - \left[\int_7^9 [h(x) - f(x)] dx \right] = - \left[\int_7^9 h(x) dx - \int_7^9 f(x) dx \right]$$

Properties
of
Definite
Integrals

$$10. \int_1^7 f(x) dx$$

$$= [4 - 5]$$

$$- [-1] \Rightarrow \textcircled{1}$$

$$\int_1^7 f(x) dx = \int_1^9 f(x) dx + \int_9^7 f(x) dx$$

$$= \underbrace{\int_1^9 f(x) dx}_{\downarrow} - \underbrace{\int_7^9 f(x) dx}_{\downarrow}$$

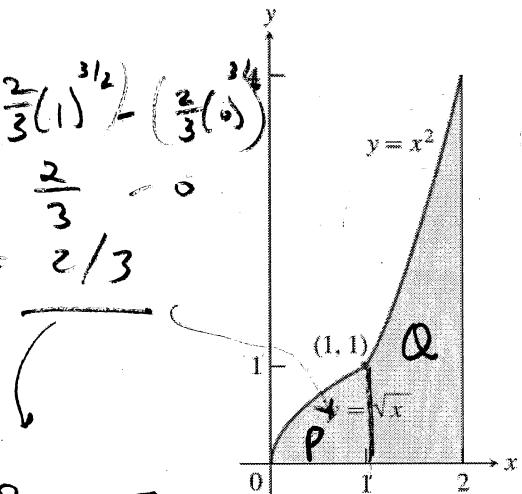
$$-1 - 5$$

$$= \textcircled{-6}$$

I-A4a

11. Find the exact area of the shaded region.

$$P: \int_0^1 \sqrt{x} dx = \int_0^1 x^{1/2} dx = \left[\frac{2}{3} x^{3/2} \right]_0^1 = \left(\frac{2}{3}(1)^{3/2} \right) - \left(\frac{2}{3}(0)^{3/2} \right)$$

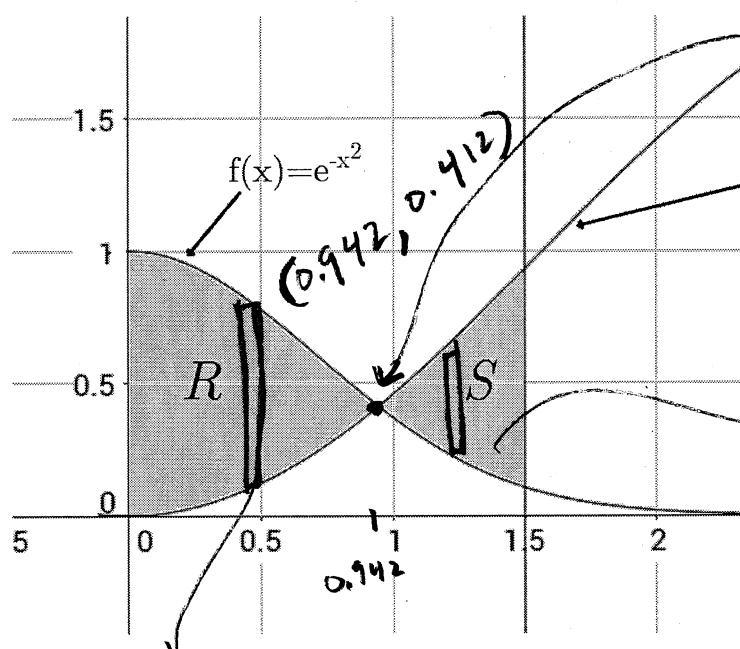


$$Q: \int_1^2 x^2 dx = \left[\frac{1}{3} x^3 \right]_1^2 \\ = \left[\frac{1}{3}(2)^3 \right] - \left[\frac{1}{3}(1)^3 \right]$$

$$\frac{8}{3} \cdot \frac{1}{3} = \frac{7}{3} \quad \rightarrow \quad \frac{2}{3} + \frac{7}{3} = \frac{9}{3} = 3$$

I-A4b

12. Let $f(x) = e^{-x^2}$ and $g(x) = 1 - \cos(x)$. The regions R and S are bounded by $f(x), g(x)$, the y-axis, and the vertical line $x = 1.5$. Find the total shaded area. Show the setup of your integrals and all related calculations.



$$y_1 = e^{-x^2}$$

$$y_2 = 1 - \cos(x)$$

Find intersection

[2nd] + [TRACE] + [5]



$$S = \int_{0.942}^{1.5} g(x) - f(x) dx$$

(top - bottom)

$$= \int_{0.942}^{1.5} 1 - \cos(x) - e^{-x^2} dx$$



0.237

$$R = \int_0^{0.942} f(x) - g(x) dx$$

(top - bottom)

$$= \int_0^{0.942} e^{-x^2} - (1 - \cos(x)) dx$$

$$= 0.591$$

$$0.591 + 0.237$$

$$= 0.828$$