I-U1

1. The Riemann definition of the definite integral is given by

$$\lim_{n\to\infty}\sum_{i=0}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$$

where $\Delta x = \frac{b-a}{n}$. Explain why this is so in the context of area under a curve. You may use diagrams to accompany your explanation.

Our interval has length $b-a$ along x-axis. This is split into n subintervals of Δx width, so $\frac{b-a}{n} = \Delta x$. We construct rectangles with ~~height~~ base Δx and a height (y-value) $f(x_i)$ where x_i is in each subinterval. Each rectangle thus has area $f(x_i)\cdot\Delta x$, and $\sum_{i=0}^{n} f(x_i)\Delta x$ is the sum of n such rectangles. As the limit where $n\to\infty$, this yields ∞ many rectangles with infinitely narrow bases, yielding the exact area under the curve.

I-U2

2. Write a definite integral whose value equals

$$\lim_{n\to\infty}\sum_{i=0}^{n} \sec^2(\underset{a}{3}+\underbrace{\frac{5i}{n}}_{\text{varies as } i\to n})\boxed{\frac{5}{n}}_{\Delta x}$$

← $b-a$ → $b-a=5$ and $a=3$

$b-3=5 \Rightarrow b=8$

$$\int_3^8 \sec^2(x)\,dx$$

3. Write the definite integral as an infinite Riemann sum using correct notation

$$\int_2^5 x^3+\sqrt{x-1}\,dx$$

$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}$

$$\lim_{n\to\infty}\sum_{i=0}^{n}\left(2+\frac{3i}{n}\right)^3\sqrt{2+\frac{3i}{n}-1}\cdot\frac{3}{n}$$

could simplify if desired.

$\sqrt{1+\frac{3i}{n}}$

$x = a+\Delta x\cdot i$

$x = 2+\frac{3i}{n}$

I-U3a

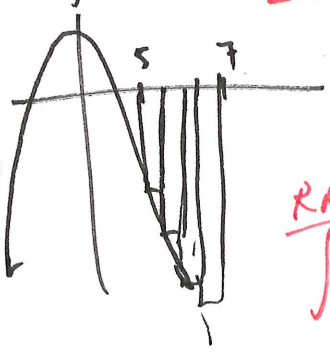
4. Find both the left and right rectangle approximations for $\int_5^7 20-(x+3)^2\,dx$ using 4 rectangles of equal width.

$f(x)$ $\qquad \Delta x = \frac{7-5}{4} = \frac{1}{2}$

LRAM RRAM

x	5	5.5	6	6.5	7
$f(x)$	-44	-52.25	-61	-70.25	-80

$\sum f(x_i)\cdot\Delta x$



LRAM

$$\int_5^7 20-(x+3)^2dx \approx \frac{1}{2}\left[-44+-52.25+-61+-70.25\right] = \frac{1}{2}\left[-227.5\right]$$

$= -113.75$

RRAM

$$\int_5^7 20-(x+3)^2dx \approx \frac{1}{2}\left[-52.25+-61+-70.25+-80\right] = \frac{1}{2}\left[-263.5\right]$$

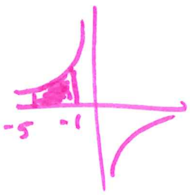
$= -131.75$

I-U3b

5. Approximate $\int_{-5}^{-1} -\frac{3}{x}\,dx$ using 4 trapezoids of equal width.

$$\Delta x = \frac{-1 - -5}{4} = \frac{4}{4} = 1$$

Trap Rule $\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2(\underbrace{\quad\quad}_{\text{all but first and last}}) + f(x_n)\right]$



$$\frac{1}{2}\left[f(-5) + 2\left[f(-4) + f(-3) + f(-2)\right] + f(-1)\right]$$

$$\frac{1}{2}\left[\frac{-3}{-5} + 2\left[\frac{-3}{-4} + \frac{-3}{-3} + \frac{-3}{-2}\right] + \frac{-3}{-1}\right]$$

$$\frac{1}{2}\left[0.6 + 2(0.75 + 1 + 1.5) + 3\right] = \frac{1}{2}(10.1) = 5.05$$

I-U3c

6. Selected values for $f(x)$ are given in the table below. Use a midpoint Riemann sum to approximate $\int_{10}^{90} f(x)\,dx$ using 4 intervals of equal width.

b, a

$$\Delta x = \frac{90-10}{4} = \frac{80}{4} = 20$$

x	10	20	30	40	50	60	70	80	90
$f(x)$	8	12	35	44	37	26	54	61	83

$$\int_{10}^{90} f(x)\,dx \approx 12 \cdot 20 + 44 \cdot 20 + 26 \cdot 20 + 61 \cdot 20$$

$$\approx 240 + 880 + 520 + 1220$$

$$\approx 2860$$

I-U5

Evaluate each definite integral.

FTC pt 2 $\int_a^b f'(x)\,dx = f(b) - f(a)$

7. $\int_{-1}^{3} x^2 - 2x\,dx$

$$\left[\frac{1}{3}x^3 - x^2\right]_{-1}^{3}$$

$$\left[\frac{1}{3}(3)^3 - (3)^2\right] - \left[\frac{1}{3}(-1)^3 - (-1)^2\right]$$

$$[9 - 9] - \left[-\frac{1}{3} - 1\right] \longrightarrow 0 - (-4/3) = 4/3$$

8. $\int_2^5 \frac{3}{x+1}\,dx$

$$3\int_2^5 \frac{1}{x+1}\,dx$$

$$3\left[\ln|x+1|\right]_2^5$$

$$3\left\{[\ln|6|] - [\ln|3|]\right\} \longrightarrow 3[\ln 6 - \ln 3] \xrightarrow{\text{properties of logs}} 3\ln 2 \longrightarrow \ln 2^3 \longrightarrow \ln 8$$

9

$\frac{d}{dt}$ $p \to v \to a$ $\int dt$

position @ $t = 3$? $\underline{x(3) = ?}$

$a(t) = -4$

$\frac{dv}{dt} = -4$

$\int dv = \int -4 \, dt$

$V = -4t + C$

<u>use information</u>: "initial velocity is -2"

$\to v(0) = -2$ *

$v(0) = -2 = -4(0) + C$

$-2 = c$

So, $v(t) = -4t - 2$

$\frac{dx}{dt} = -4t - 2$

$\int dx = \int -4t - 2 \, dt$

$x(t) = -2t^2 - 2t + C$

<u>use information</u>: "Initial position is 12 m"

$\to x(0) = 12$ *

$x(0) = 12 = -2(0)^2 - 2(0) + c$

$12 = c$

$\to$ So, $x(t) = -2t^2 - 2t + 12$

<u>$x(3)$?</u>

$x(3) = -2(3)^2 - 2(3) + 12$

$-18 - 6 + 12$

$= -12$ meters