

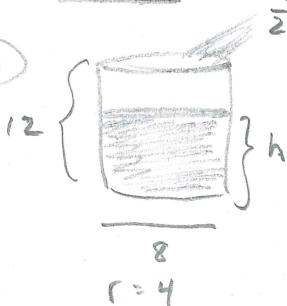
D-AD15

Practice Assessment

1. Water is being pumped into a 12-foot-tall cylindrical tank with diameter 8 feet. It is being pumped at a constant rate of 0.5 cubic feet per second. At what rate is the depth of the water changing? Include units in your answer.

$$\frac{dV}{dt} = \frac{1}{2}$$

$$\frac{dh}{dt} = ?$$

DREDS

* Note: Diameter/radius
is constant! *

(R)

$$\frac{dV}{dt} = \frac{1}{2}$$

$$\frac{dh}{dt} = ?$$

(E)

$$V = \pi r^2 h$$

$$V = \pi \cdot 4^2 \cdot h$$

$$V = 16\pi \cdot h$$

(D)

$$V = 16\pi h$$

$$\frac{dV}{dt} = 16\pi \frac{dh}{dt}$$

(S)

$$2 \left(\frac{1}{2} = 16\pi \cdot \frac{dh}{dt} \right) ?$$

$$1 = 32\pi \frac{dh}{dt}$$

$$\frac{1}{32\pi} \text{ ft/sec} = \frac{dh}{dt}$$

I-A3

2. The acceleration of a horizontally moving particle can be modeled by the function $a(t) = -3t \text{ ft/sec}^2$. Find the position of the particle at $t=4$ if the initial velocity is 12 ft/sec and initial position is -2 ft . Include units in your answer.

$$p(4) = ?$$

$$v(0) = 12$$

$$p(0) = -2$$

$$\frac{d}{dt}(P \uparrow V) \\ \frac{d}{dt}(a) \uparrow$$

$$a(t) = \frac{dv}{dt} = -3t$$

$$dv = -3t dt$$

$$\int dv = \int -3t dt$$

$$v(t) = -\frac{3}{2}t^2 + C$$

use $v(0) = 12$

$$v(0) = 12 = -\frac{3}{2} \cdot 0^2 + C$$

$$C = 12$$

$$v(t) = -\frac{3}{2}t^2 + 12$$

$$v(t) = \frac{dP}{dt} = -\frac{3}{2}t^2 + 12$$

$$\int dP = \int -\frac{3}{2}t^2 + 12 dt$$

$$P(t) = -\frac{3}{2} \cdot \frac{t^3}{3} + 12t + C$$

$$P(t) = -\frac{1}{2}t^3 + 12t + C$$

use $P(0) = -2$

$$P(0) = -2 = -\frac{1}{2} \cdot 0^3 + 12(0) + C$$

$$-2 = C$$

$$P(t) = -\frac{1}{2}t^3 + 12t - 2$$

$$P(4) = ?$$

$$P(4) = -\frac{1}{2}(4)^3 + 12(4) - 2$$

$$= -32 + 48 - 2$$

$$= 14 \text{ feet}$$

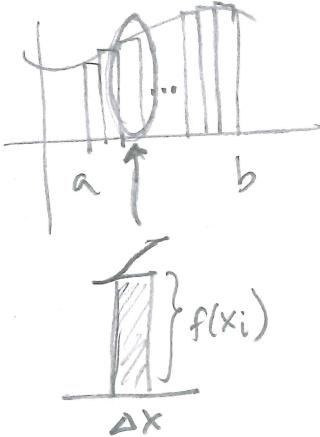
when!

I-U1

3. The Riemann definition of the definite integral is given by

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_a^b f(x) dx$$

where $\Delta x = \frac{b-a}{n}$. Explain why this is so in the context of area under a curve. You may use diagrams to accompany your explanation.



Interval size is $b-a$. Split into n subintervals, called Δx . Rectangles have Δx as base and $f(x_i)$ as height, giving $f(x_i)\Delta x$ as area. $\sum_{i=1}^n$ gives sum of n -rectangle's area. To give exact area, increase $n \rightarrow \infty$ so each rectangle is thin and there are ∞ many of them. This gives the exact area, $\int_a^b f(x) dx$.

I-U2

4. Write a definite integral whose value equals

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin\left(2 + \frac{4i}{n}\right) \frac{4}{n} \Delta x$$

$$\begin{aligned} \frac{4}{n} &= \frac{b-a}{n} = \Delta x \\ 4 &= b-a \\ 4 &= b-2 \\ 6 &= b \end{aligned}$$

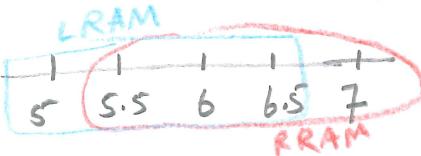
$$\int_a^b f(x) dx$$

Need these 3

$$\int_2^6 \sin(x) dx$$

I-U3a

5. Find both the left and right rectangle approximations for $\int_5^7 -(x+3)^2 + 20 dx$ using 4 rectangles of equal width.



$$\begin{aligned} n &= 4 \\ \Delta x &= \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2} \end{aligned}$$

LRAM

$$\begin{aligned} \int_5^7 -(x+3)^2 + 20 dx &\approx \frac{1}{2} [f(5) + f(5.5) + f(6) + f(6.5)] \\ &\approx \frac{1}{2} [-44 + -52.25 + -61 + -70.25] \\ &= \frac{1}{2} [-227.5] \\ &\approx -113.75 \end{aligned}$$

RRAM

$$\begin{aligned} &\approx \frac{1}{2} [f(5.5) + f(6) + f(6.5) + f(7)] \\ &= \frac{1}{2} [-52.25 + -61 + -70.25 + -80] \\ &= \frac{1}{2} [-263.5] \end{aligned}$$

$$= -131.75$$

I-U3b

R

I-U3b

n=4

$$\frac{\Delta x}{2} [f(a) + 2(f(x_1) + \dots + f(x_{n-1})) + f(b)]$$

6. Approximate $\int_{-5}^{-1} -\frac{3}{x} dx$ using 4 trapezoids of equal width.

$$\Delta x = \frac{-1 - (-5)}{4} = \frac{-1 + 5}{4} = \frac{4}{4} = 1$$

$$\frac{1}{2} [f(-5) + 2[f(-4) + f(-3) + f(-2)] + f(-1)]$$

$$\frac{1}{2} [-\frac{3}{5} + 2[-\frac{3}{4} + -\frac{3}{3} + -\frac{3}{2}] + -\frac{3}{1}]$$

$$\frac{1}{2} [0.6 + 2(0.75 + 1 + 1.5) + 3] \Rightarrow \frac{1}{2} (10.1) = \boxed{5.05}$$

I-U3c

7. Selected values for $f(x)$ are given in the table below. Use a midpoint Riemann sum to approximate $\int_{10}^{90} f(x) dx$ using 4 intervals of equal width.

| | | | | | | | | | |
|--------|----|----|----|----|----|----|----|----|----|
| x | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| $f(x)$ | 8 | 12 | 35 | 44 | 37 | 26 | 54 | 61 | 83 |

$$\Delta x = \frac{b-a}{n} = \frac{90-10}{4} = \frac{80}{4} = 20$$

$\underbrace{20}_{20} \quad \underbrace{20}_{20} \quad \underbrace{20}_{20} \quad \underbrace{20}_{20}$

$$\begin{aligned} \int_{10}^{90} f(x) dx &\approx 12(20) + 44(20) + 26(20) + 61(20) \\ &\approx 240 + 880 + 520 + 1220 \\ &= \boxed{2860} \end{aligned}$$

I-U5

8. $\int_{-1}^3 x^2 - 2x dx$

$$\left[\frac{1}{3}x^3 - x^2 \right]_{-1}^3$$

$$\left[\frac{1}{3} \cdot 3^3 - 3^2 \right] - \left[\frac{1}{3}(-1)^3 - (-1)^2 \right] \Rightarrow [9-9] - \left[-\frac{1}{3} - 1 \right]$$

$$f(b) - f(a) \quad 0 - -\frac{4}{3} = \boxed{\frac{4}{3}}$$

9. $\int_2^5 \frac{3}{x+1} dx$

$$= \int_2^5 3 \cdot \frac{1}{x+1} dx$$

$$3 \int_2^5 \frac{1}{x+1} dx$$

$$3 \left[\ln|x+1| \right]_2^5 \Rightarrow 3 \left[\ln|5+1| - \ln|2+1| \right] \Rightarrow 3 \left[\ln 6 - \ln 3 \right] \rightarrow$$

$$\ln 8 \quad \leftarrow \ln 2^3 \quad \leftarrow 3 \ln 2 \quad \leftarrow 3 \cdot \ln \frac{6}{3}$$

FTC 2

$$\int_a^b f'(x) dx = [f(x)]_a^b = f(b) - f(a)$$

[Log Rules]

$$a \cdot \ln b \Leftrightarrow \ln b^a$$

$$\ln b + \ln a \Leftrightarrow \ln(b \cdot a)$$

$$\ln b - \ln a \Leftrightarrow \ln \frac{b}{a}$$