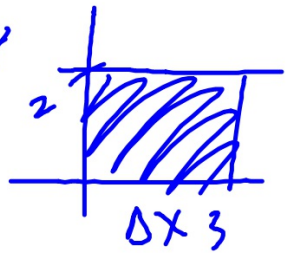
AP Calculus AB

Every year, a stone mysteriously rolls 2 feet farther down a path in Death Valley, CA. How far did it roll between 1993 and 1996? Justify your answer.

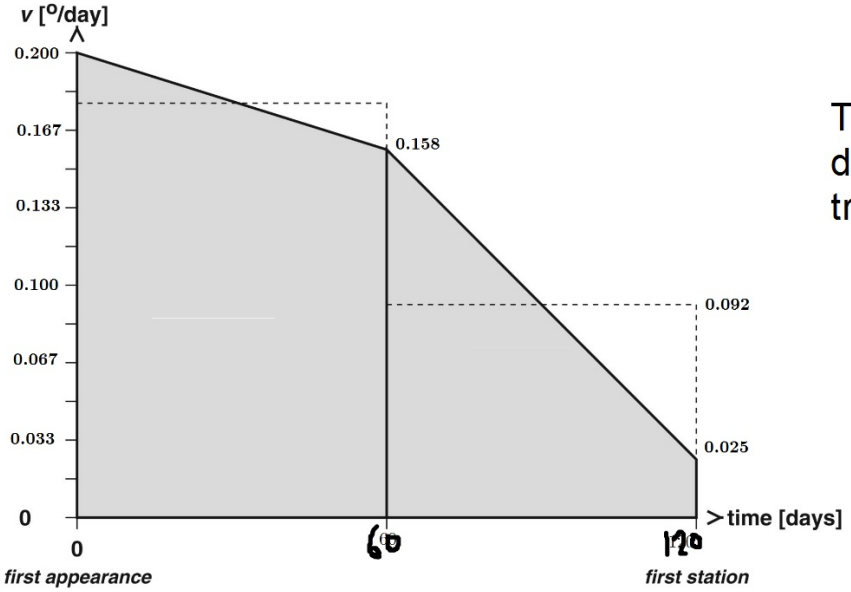
$D = r \cdot t$ $[1993, 1996]$

$\frac{2\text{ ft}}{\text{yr}} \cdot 3\text{yr}$

6 ft



Ancient Babylonians measured the change in location of Jupiter over 120 days. They measured the change in degrees per day along its path in the sky. Here is their data transcribed from their Cuneiform tablets dating back to ~300BCE.



Through how many degrees did Jupiter travel after 120 days?

Central questions: How do you find the area of weird shaped spaces?
What does it mean to "accumulate"?
What does "net change" mean?

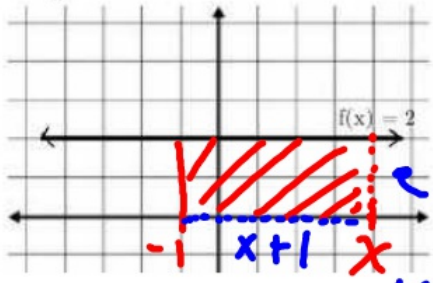
Questions from last class' discussion on finding area, approximations, and definite integrals?

Please get out your 2-pg packet on area/definite integrals. Let's look back at pg 3

Connection between Area and Antiderivatives and Slope

For each function, use geometry to find the area A(x) *under the function* f(x) *between* -1 and some arbitrary point x (or, over the interval [-1,x]). *Then, find* A'(x). What do you notice about f(x) and A'(x)?

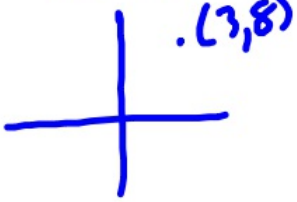
1.



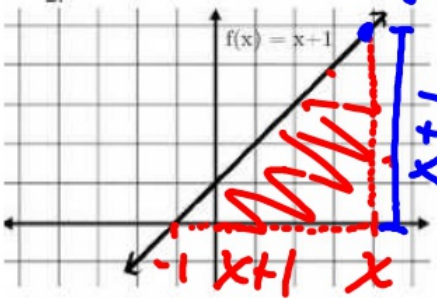
f(x) = 2

Area function A(x) = $2(x+1) = 2x+2$

A'(x) or $\frac{dA}{dx}$ = 2



2.



f(x) = x+1 $(x, x+1)$

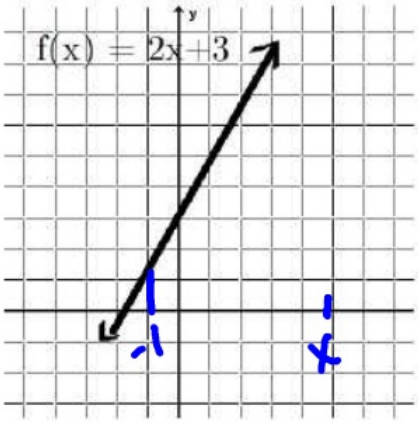
Area function A(x) = $\frac{1}{2}(x+1)(x+1)$

$\frac{1}{2}(x^2+2x+1) =$

$A(x)\ \frac{1}{2}x^2 + x + \frac{1}{2}$

A'(x) or $\frac{dA}{dx}$ =

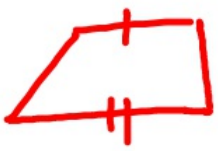
3.



f(x) = 2x+3 $\int 2x+3\,dx$

Area function A(x) =

$A_{trap} = \frac{1}{2}(b_1+b_2)\cdot h$



A'(x) or $\frac{dA}{dx}$ =

Antideriv.

f(x) → Area under f(x) A(x)

deriv.

Now go back and find the area under the curve using the FTC:

1. $\int_{-1}^{x} 2\,dx$

2. $\int_{-1}^{x} x+1\,dx$

3. $\int_{-1}^{x} 2x+3\,dx$

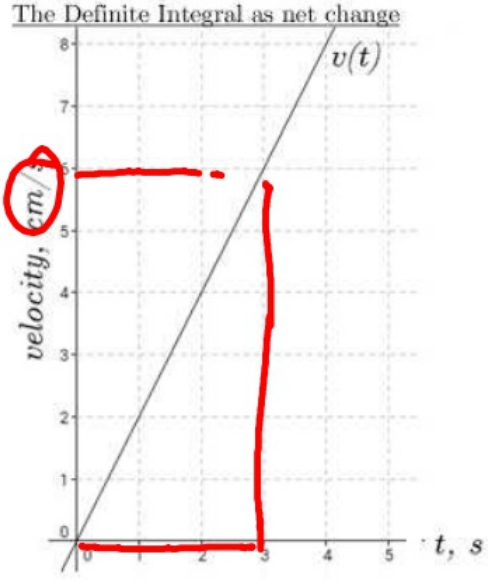
The Fundamental Theorem of Calculus: Part 2

If $f(x)$ is the derivative of $F(x)$, then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

For real numbers a and b (called the limits of integration). It is not required that $a < b$.

The Definite Integral as net change



An object moves along the x-axis such that its velocity in cm/s is given by $v(t) = 2t$. At time $t = 0\,s$, the object is at the origin. After 3 seconds, how far as the object traveled?

1. Find the specific position function $x(t)$.

$x(t) = t^2 + C$

$0 = 0^2 + C \Rightarrow C = 0$

(0,0)

$x(t) = t^2$

2. Use the position function to find the difference between the positions ("displacement") at time $t = 3$ and time $t = 0$.

$x(3) = 3^2 = 9$

$x(0) = 0^2 = 0$ $= 9 - 0 = 9\,cm$

3. Find the exact area (using geometry) under the velocity function in the same time interval as problem 2. Use units in your calculations.

$A = \frac{1}{2} \cdot 3\,sec \cdot 6\,\frac{cm}{sec} = 9\,cm$

4. Write a *definite* integral that will find the displacement. Then use the second FTC to evaluate the integral.

$$\int_0^3 2t\,dt = \left[t^2\right]_0^3 = 3^2 - 0^2 = 9$$

5. In a complete sentence, write a conjecture about what you think the definite integral can be used to find.

It's winter and snow is falling at 12 noon (t=0). Snow rate measurements were made on the hour. At noon, snow fell at a rate of 0.5 cm/hr. At 1pm, snow fell at a rate of 1 cm/hr. Further measurements are recorded here.

time	Noon (0)	1pm (1)	2pm (2)	3pm (3)	4pm (4)	5pm (5)
cm/hr	0.5	1	1.5	1.5	2	0.5

How much snow is on the ground at 5pm?