Good afternoon: warm up

An object is moving such that its velocity is modeled by $v(t)=t^2+1$ m/s. Find the position of the object at $t=5$ if at $t=1$, the object's position is 5 meters from the origin.

$X(5) = ?$

$X(1) = 5$

P

V

a

$\cancel{v(t)} = t^2 + 1$

$\frac{dx}{dt} = t^2 + 1$

$\int dx = \int t^2 + 1 \, dt$ (Rev. Power)

$x = \frac{1}{3}t^3 + t + C$

$5 = \frac{1}{3} + 1 + C$

$\frac{11}{3} = C$

$x(t) = \frac{1}{3}t^3 + t + \frac{11}{3}$

$X(5) = \boxed{50\tfrac{1}{3}\text{ m}}$

Next assessment: Monday

same 3 topics as last test (no Riemann sums yet)

HW answers

25 LRAM 13, RRAM 15

26 LRAM 37/3, RRAM 35/3 (12.333, 11.667)

27 LRAM 55, RRAM 74.5

28 LRAM 155/16, RRAM 187/16 (9.6875, 11.6875)

29 LRAM 1.184 RRAM 0.791

30 LRAM and RRAM both 1.941

33 LRAM: 0.518, RRAM: 0.768

34 LRAM: 5.685, RRAM: 6.038

Understand calculus concepts...

...verbally

...numerically

...algebraically

...graphically

Be fluent algebraically:

Write a definite integral whose value equals

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\overset{a}{3}+\frac{5}{n}i\right)^3-\left(3+\frac{5}{n}i\right)^2-4\right]\frac{5}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\boxed{\Delta x} = \int_a^b f(x)\,dx$$

$$\Delta x = \frac{b-a}{n} = \frac{5}{n} \leftarrow$$

$$a = 3 \longrightarrow b = 8$$

$$\boxed{\int_3^8 \underbrace{x^3 - x^2 - 4}\,dx}$$

Write a definite integral whose value equals

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(6+\frac{2i}{n}\right)\frac{2}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)\,dx$$

$$\Delta x = \frac{b-a}{n}$$

$$\int_{6}^{8} \sin(x)\,dx$$

"the definite integral from 6 to 8 of sin(x)."

Write a infinite Riemann sum whose value equals

$$\int_5^7 \sqrt{x+3}\,dx$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)\,dx$$

$$\Delta x = \frac{b-a}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \underline{\sqrt{\left(5+\frac{2}{n}i\right)+3}} \cdot \underline{\frac{2}{n}}$$

"$a + \Delta x \cdot i$"

Write a infinite Riemann sum whose value equals

$$\int_2^9 x^2 + 2x \, dx$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(2+\frac{7}{n}i\right)^2 + 2\left(2+\frac{7}{n}i\right) \cdot \frac{7}{n}$$

$$\int_2^7 e^{3x} dx$$

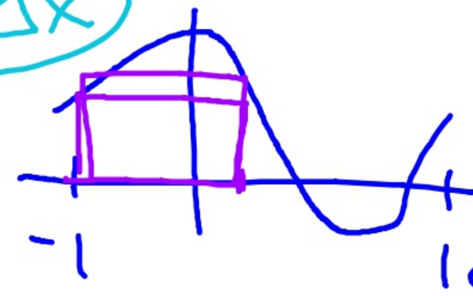
$$\lim_{n\to\infty} \sum_{i=1}^{n} e^{3\left(2+\frac{5}{n}i\right)} \cdot \frac{5}{n}$$

Numerical Fluency:

Suppose f(x) is a differentiable function. Here are some values:

$\Delta x$

| x | -1 | 2 | 3 | 5 | 7 | 10 |
|---|---|---|---|---|---|---|
| f(x) | 9 | 10 | 2 | -1 | -3 | 3 |



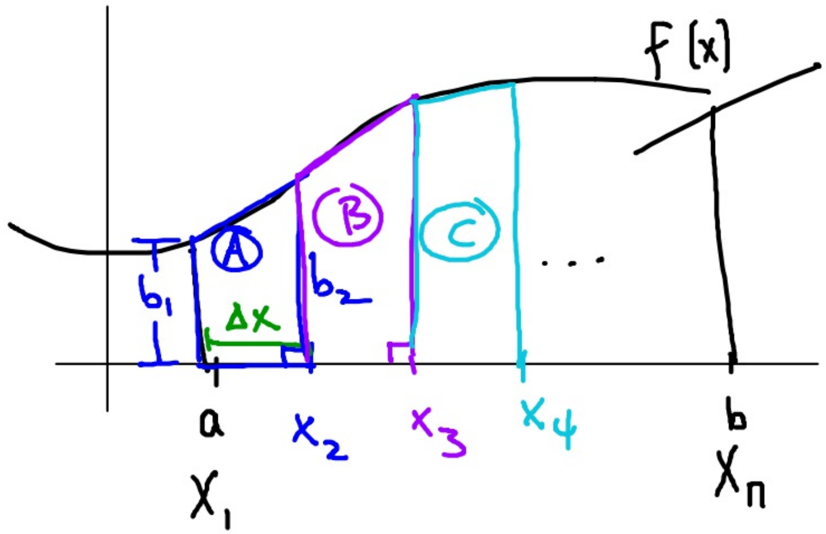

Estimate $\int_{-1}^{10} f(x)\,dx$ using a right Riemann sum.

$$\approx 10(3) + 2(1) + -1(2) + -3(2) + 3(3)$$

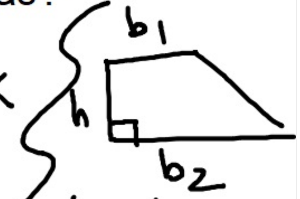
$$30 + 2 - 2 - 6 + 9$$

$$33$$

LRAM, RRAM, MRAM....is there a better way to approximate exact areas?




$$= \int_a^b f(x)\,dx$$




$$= \frac{b_1+b_2}{2}\cdot h$$

$$= (b_1+b_2)\cdot\frac{h}{2}$$

TRAP. A

$$\approx (f(x_1)+f(x_2))\cdot\frac{\Delta x}{2}$$

Trap B

$$+ (f(x_2)+f(x_3))\cdot\frac{\Delta x}{2}$$

Trap C

$$+ (f(x_3)+f(x_4))\cdot\frac{\Delta x}{2} + \dots$$

$$\frac{\Delta x}{2}\cdot\left(f(x_1)+f(x_2)+f(x_2)+f(x_3)+f(x_3)+f(x_4)+\dots\right)$$

$$\frac{\Delta x}{2}\left(f(x_1)+2\left[f(x_2)+f(x_3)+\dots f(x_{n-1})\right]+f(x_n)\right)$$

trapezoid rule

Approximate $\int_4^7 e^{2x} \cdot dx$ using 6 trapezoids.

$n = 6$

$\Delta x = \frac{3}{6} = \frac{1}{2}$

$$\approx \frac{\Delta x}{2}\left[f(x_1) + 2\left(f(x_2) + f(x_3) + \ldots\right) \ldots + f(x_n)\right]$$

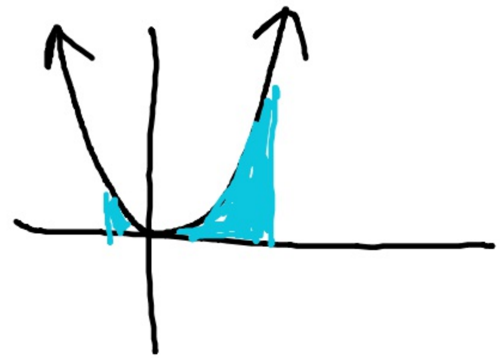
$$\cdot \frac{1/2}{2}\left[f(4) + 2\left(f(4.5) + f(5) + f(5.5) + f(6) + f(6.5)\right) + f(7)\right]$$

$$\frac{1}{4}\left[e^{8} + 2\left(e^{9} + e^{10} + e^{11} + e^{12} + e^{13}\right) + e^{14}\right]$$

Approximate $\int_{-1}^{3} x^2\,dx$ using 4 trapezoids.

$n$

$$\Delta x = \frac{3--1}{4} = \frac{4}{4} = 1$$



$$\frac{\Delta x}{2}\left[f(x_1)+2(\cdots)+f(x_n)\right]$$

$$\frac{1}{2}\left[f(-1)+2\left(f(0)+f(1)+f(2)\right)+f(3)\right]$$

$$\frac{1}{2}\left[1+2(0+1+4)+9\right] = 10$$

Determining over- vs underestimate with integral approximation methods

t.b.d.

Putting it all together

$$\left(\frac{b_1+b_2}{2}\right)\cdot h$$

| x | -5 | -3 | 0 | 1 | 5 |
|---|---|---|---|---|---|
| f(x) | 10 | 7 | 5 | 8 | 11 |

| $x$ | $-5$ | $-3$ | $0$ | $1$ | $5$ |
|---|---|---|---|---|---|
| $f(x)$ | 10 | 7 | 5 | 8 | 11 |

Given the values for $f(x)$ on the table above, approximate the area under the graph of $f(x)$ from $x=-5$ to $x=5$ using four subintervals and a **Trapezoidal approximation.**


$$\left(\frac{10+7}{\cancel{2}}\right)\cdot\cancel{2} + \left(\frac{7+5}{2}\right)3 + \left(\frac{5+8}{2}\right)1 + \left(\frac{8+11}{\cancel{2}}\right)\cdot\cancel{4}\,2$$

$$17 + 18 + 6.5 + 38$$

$$= 79.5$$

HW
handout #1-12
number answers posted to mcalc.weebly.com