Good afteroon: warm up

$$5\int \frac{\cancel{5}x^2}{4x^3+2}\,dx = \frac{5}{12}\int \frac{12}{5}\cancel{5}x^2 \cdot \frac{1}{4x^3+2}\,dx$$

want: $12x^2$

$$\frac{5}{12}\int \cdot \frac{1}{4x^3+2}\,dx$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\boxed{\frac{5}{12}\ln|4x^3+2| + C}$$

$$\frac{5}{\cancel{12}} \cdot \frac{1}{4x^3+2} \cdot \cancel{12}x^2$$

$$\frac{5x^2}{4x^3+2}.$$

visibly random grouping

HW answers....

just check your work by taking the derivative!

862.) $\int \frac{1}{x^3}\,dx \longrightarrow \int x^{-3}\,dx$

$\boxed{-\frac{1}{2}x^{-2}+C}$

$\int \frac{1}{\sin(x)} \cdot \cancel{\cos(x)}\,dx$

$\ln|\sin(x)|+C$

$\int \frac{1}{\sin^2(x)}\cos(x)\,dx$

864) $\int x^2 \cdot \sqrt{x}\ dx$

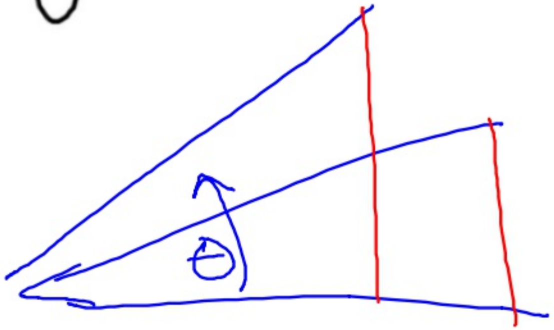
$$\int x^{2} \cdot x^{\frac{1}{2}}\ dx$$

$$\int x^{\frac{5}{2}}\ dx \rightarrow \boxed{\frac{2}{7}x^{7/2} + C}$$

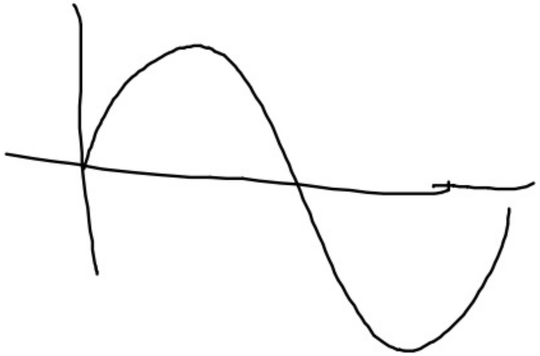
Reverse chain Rules cont'd.

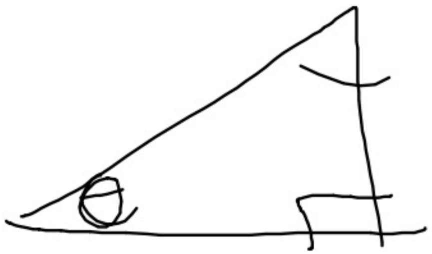
$$\int \cos(x)\cdot \sin^2(x)\,dx = \int \cos(x)\cdot \left[\underline{\sin(x)}\right]^2 dx$$

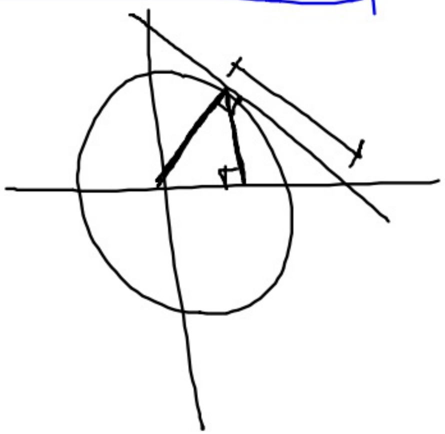
want: cos(x)



$$\boxed{\frac{1}{3}\sin^3(x) + C}$$







$$\int \frac{3x}{\sqrt{1-16x^4}}\,dx = \frac{3}{8}\int \frac{8}{3}\,3x \cdot \frac{1}{\sqrt{1-(4x^2)^2}}\,dx$$

want: $8x$

$$= \frac{1}{x^2}\int x^2\, 3x\,(1-16x^4)^{-1/2}\,dx$$

want: $-64x^3$

$$\frac{3}{8}\int 8x \cdot \frac{1}{\sqrt{1-(4x^2)^2}}\,dx$$

$$\boxed{\frac{3}{8}\arcsin(4x^2)+C}$$

$$\frac{3}{8}\cdot\frac{1}{\sqrt{1-(4x^2)^2}}\cdot 8x$$

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arccos = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arctan = \frac{1}{1+x^2}$$

What to do when the reverse chain rule fails?

$$\int x \cdot \sqrt{2x-1} \cdot dx$$

$$\int x \cdot (2x-1)^{1/2} \cdot dx$$

want: 2

Can't use (reverse chain rule)

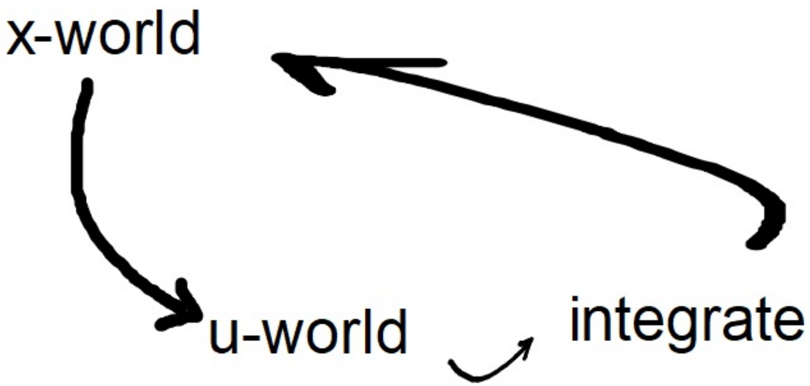
note, u-sub can also solve any problem that the reverse chain rule can do.

Basic Premise

Shift from an 'x' problem to a 'u' problem, recalibrating the question

1. Assume new variable u is something based on x
2. Based on assumption, make 2 new deductions for x and dx
3. Rewrite original in terms of u and du, not x and dx

Solve the easier 'u' problem, then reshift back to the x variable



$$\int x\sqrt{2x-1}\,dx$$

$$\int x(2x-1)^{1/2}\,dx$$

Let $u=2x-1$.

then $x=\frac{u+1}{2}$

also, $\frac{du}{dx}=2 \longrightarrow du=dx\cdot 2 \longrightarrow \frac{1}{2}du=dx$

deduce statements for x and dx from assumption.

$\frac{d}{dx}u=\frac{d}{dx}2x-1$

$\frac{du}{dx}=2$

$$\int \frac{u+1}{2}\cdot(u)^{1/2}\cdot\frac{1}{2}du$$

$$\int \frac{1}{2}(u+1)u^{1/2}\cdot\frac{1}{2}\cdot du$$

$$\frac{1}{4}\int(u+1)u^{1/2}\cdot du=\frac{1}{4}\int u^{3/2}+u^{1/2}\,du$$

$$\frac{1}{4}\left[\frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}+C\right]$$

$u=2x-1$

$$\frac{2}{20}u^{5/2}+\frac{2}{12}u^{3/2}+C$$

$$\boxed{\frac{1}{10}(2x-1)^{5/2}+\frac{1}{6}(2x-1)^{3/2}+C}$$

$$\int \sqrt{4x^2-3} \cdot x \, dx$$

Bonus!

Doesn't Require u-sub, but:

$$\int x \cdot (4x^2-3)^{1/2} \cdot du$$

Let $u = 4x^2-3$

then: $\frac{du}{dx} = 8x \rightarrow \frac{du}{8x} = dx$

Tip: when using u-sub in place of reverse chain rule, DO NOT solve for X.

Sub:

$$\int \cancel{x} \cdot u^{1/2} \cdot \frac{du}{8\cancel{x}}$$

$$\frac{1}{8}\int u^{1/2} \cdot du \longrightarrow \frac{1}{8} \cdot \left[\frac{2}{3}u^{3/2} + C\right] \rightarrow \frac{1}{12}u^{3/2} + C$$

$$\boxed{\frac{1}{12}(4x^2-3)^{3/2} + C}$$

$$\int \frac{3x}{4x-2}\,dx$$

$$\int 3x\,\frac{1}{4x-2}\,dx$$

Let $u = 4x-2$

$x = \frac{u+2}{4} = \frac{1}{4}(u+2)$

$\frac{du}{dx} = 4 \to dx = \frac{1}{4}du$

$$\int 3\cdot\frac{1}{4}(u+2)\,\frac{1}{u}\cdot\frac{1}{4}\,du$$

$$\frac{3}{16}\int (u+2)\,u^{-1}\cdot du$$

$$\frac{3}{16}\int 1 + 2u^{-1}\cdot du$$

$$\frac{3}{16}\int 1 + \frac{2}{u}\,du$$

$\int \frac{1}{u}\,du = \ln|u| + C$

$u^{1}\cdot u^{-1}$

$\frac{u^{1}}{u^{1}}$

$$\frac{3}{16}\left[u + 2\cdot\ln|u| + C\right]$$

$$\frac{3}{16}u + \frac{3}{8}\ln|u| + C$$

$$\boxed{\frac{3}{16}(4x-2) + \frac{3}{8}\ln|4x-2| + C}$$

$$\int \frac{3x}{\sqrt{4x+1}}\,dx$$

What's on Thursday's Test:

Basic Antiderivatives
Reverse Chain Rule
U-Substitution

$$= \int 3x \cdot (4x+1)^{-1/2} \cdot dx$$

$$\boxed{u = 4x+1} \longrightarrow \boxed{\tfrac{1}{4}(u-1) = x}$$

$$\frac{du}{dx} = 4 \longrightarrow \boxed{\tfrac{1}{4}du = dx}$$

$$\int 3 \cdot \tfrac{1}{4}(u-1) \cdot (u)^{-1/2} \cdot \tfrac{1}{4}du$$

$$\frac{3}{16}\int u^{\frac{1}{2}} - u^{-1/2} \cdot du \longrightarrow \frac{3}{\cancel{16}_{8}}\left[\frac{\cancel{2}}{3}u^{3/2} - 2u^{\frac{1}{2}} + C\right]$$

$$\boxed{\frac{3}{24}(4x+1)^{3/2} - \frac{3}{8}(4x+1)^{1/2} + C} \leftarrow \frac{3}{24}u^{3/2} - \frac{3}{8}u^{\frac{1}{2}} + C$$

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