Good afternoon and welcome back!

Warm up

Find the area between f(x)=sin(x) and g(x)=cos(x) over [-3π/4, 5π/4] without a calculator

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin x - \cos x \; dx$$

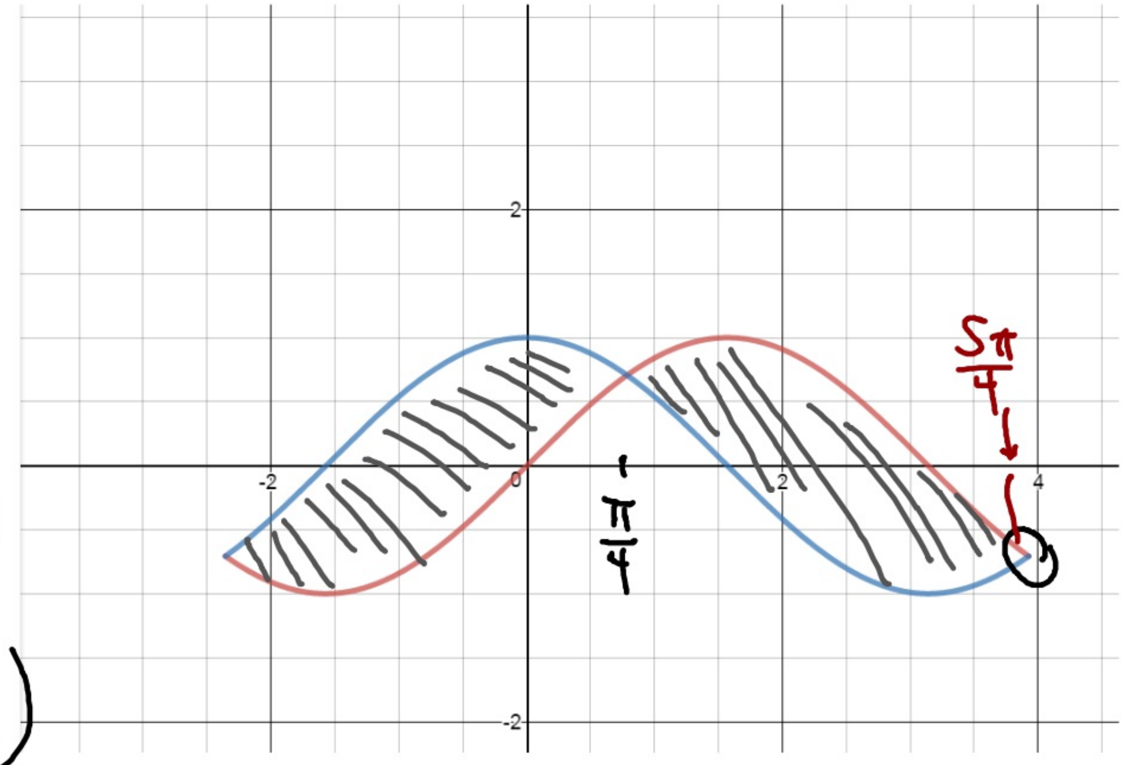
$$\left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$\left(-\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4}\right) - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$$

$$\left(+\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$\sqrt{2} \; - \; -\sqrt{2}$$

$$2\sqrt{2} \times 2 \rightarrow 4\sqrt{2}$$



visibly random grouping

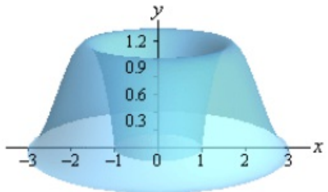
What are we doing this quarter:

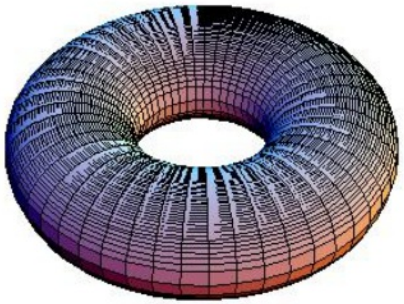
- volume
- average value
- differential equations/slope fields
- AP test review

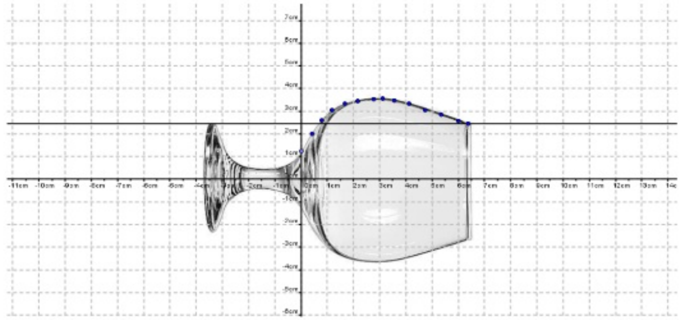
When is senior moratorium day?

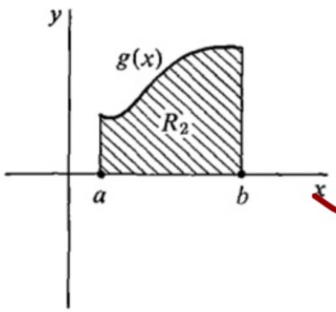
May 6 (?)

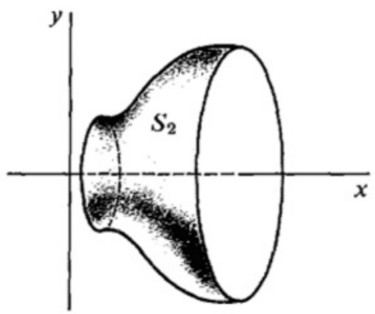
Volume of Revolution

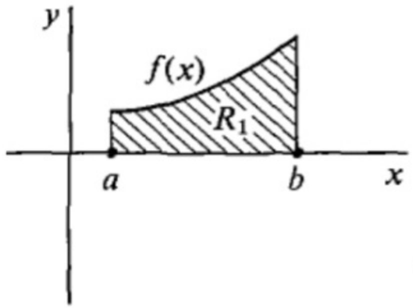


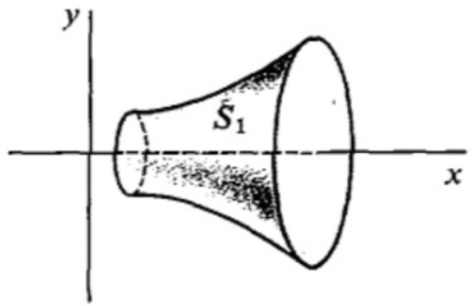


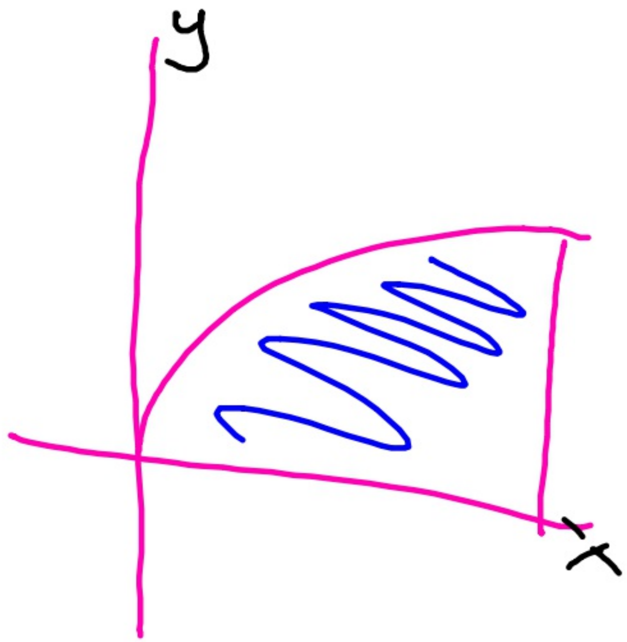










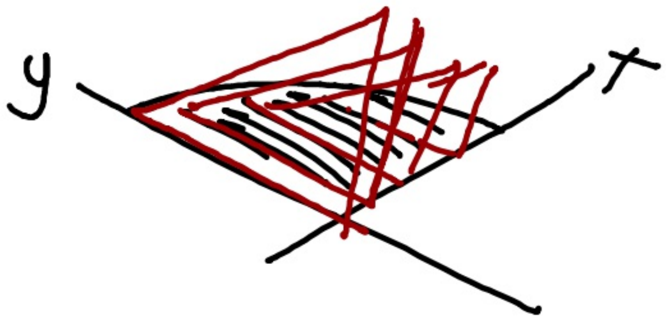


we will <u>later</u> study volume by cross section

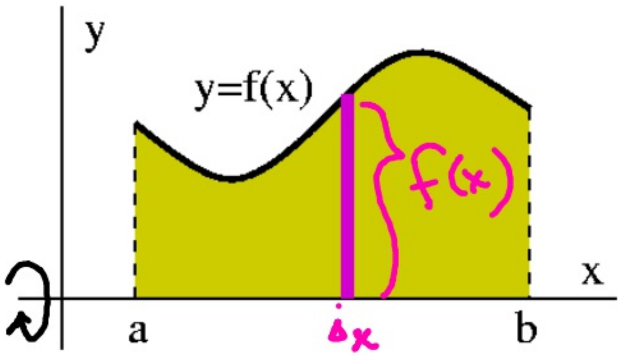
here, the region is the base of a shape

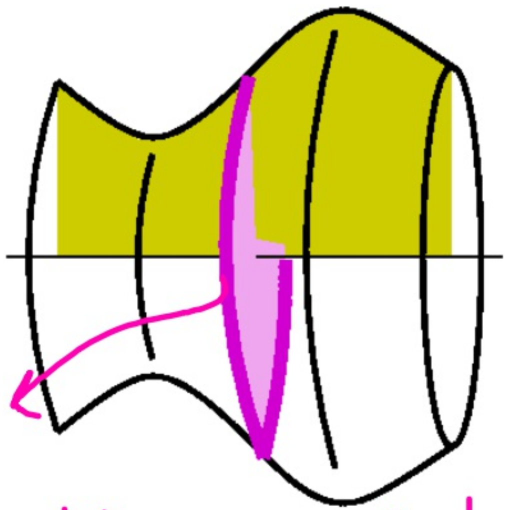
with specified shapes for slices

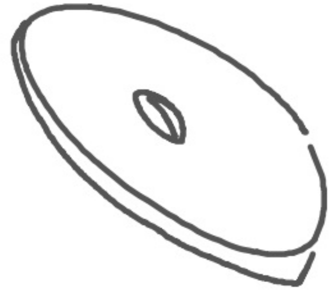
it is NOT revolved



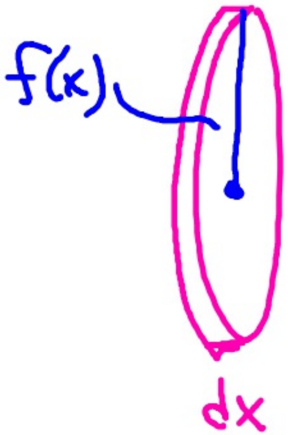
Volume of Solids of Revolution (disk method and washer method)







$\int$



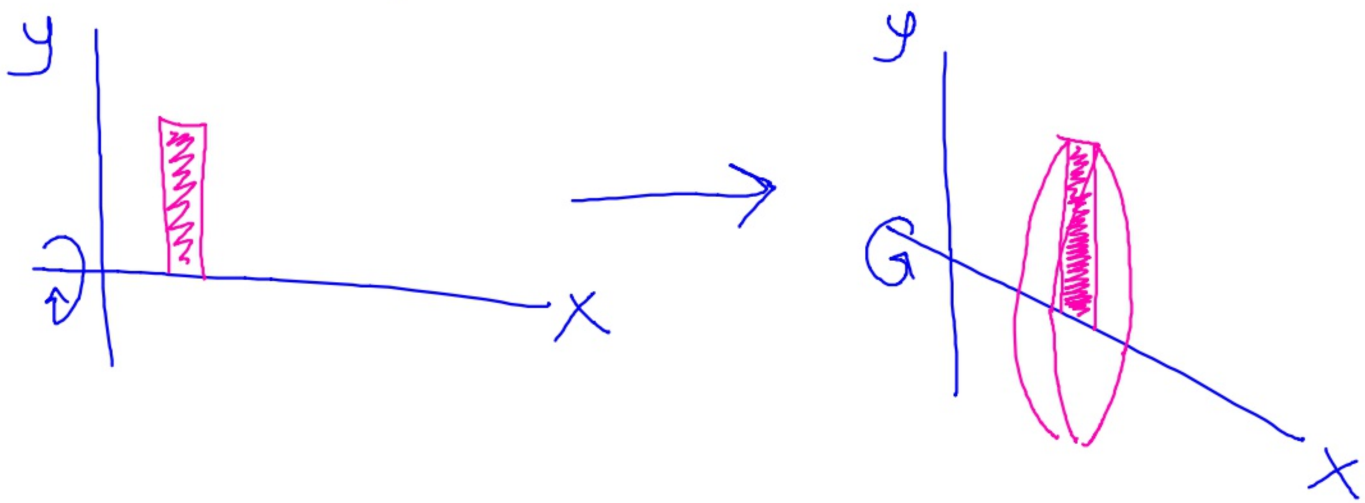
$V_{cyl} = \pi r^2 \cdot h$

$\pi [f(x)]^2 dx$ ← volume of single disk

$$V = \int_a^b \pi (f(x))^2 \cdot dx$$

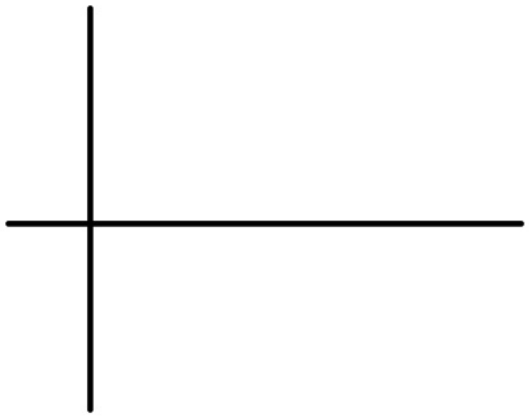
What happens when you revolve a rectangle around an axis?

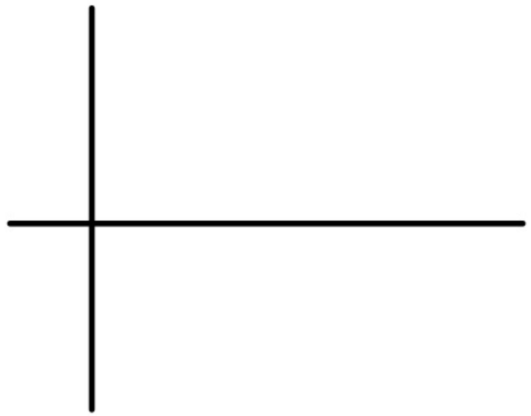
becomes a cylinder



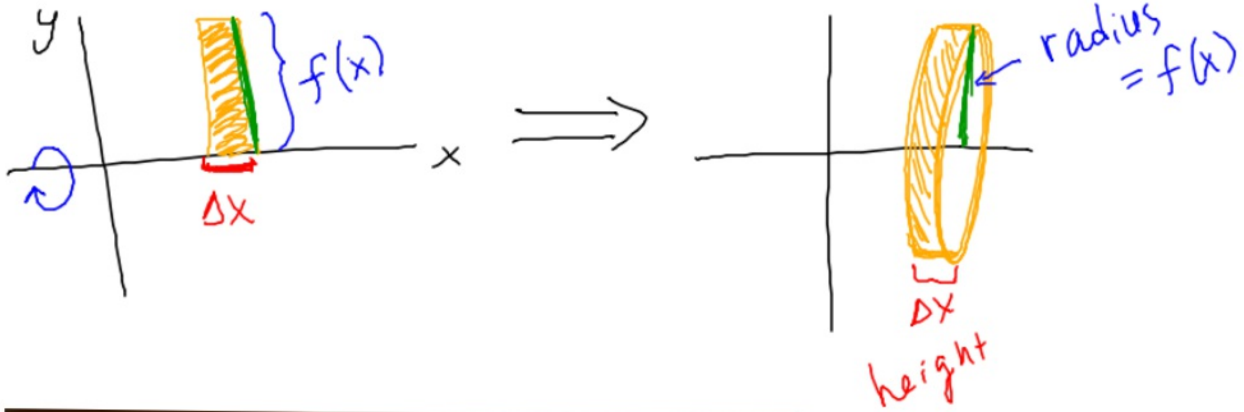
So a single rectangle becomes a cylindrical "disk"

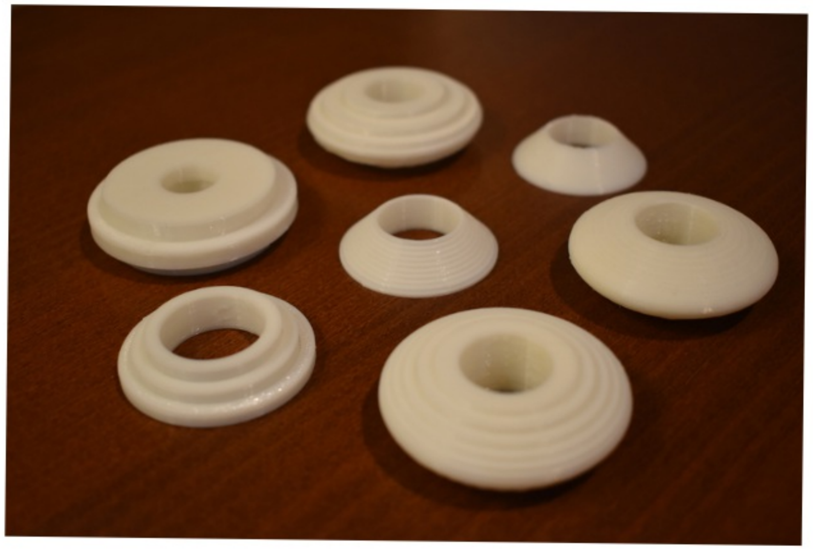
What is its radius? What is its height/depth?



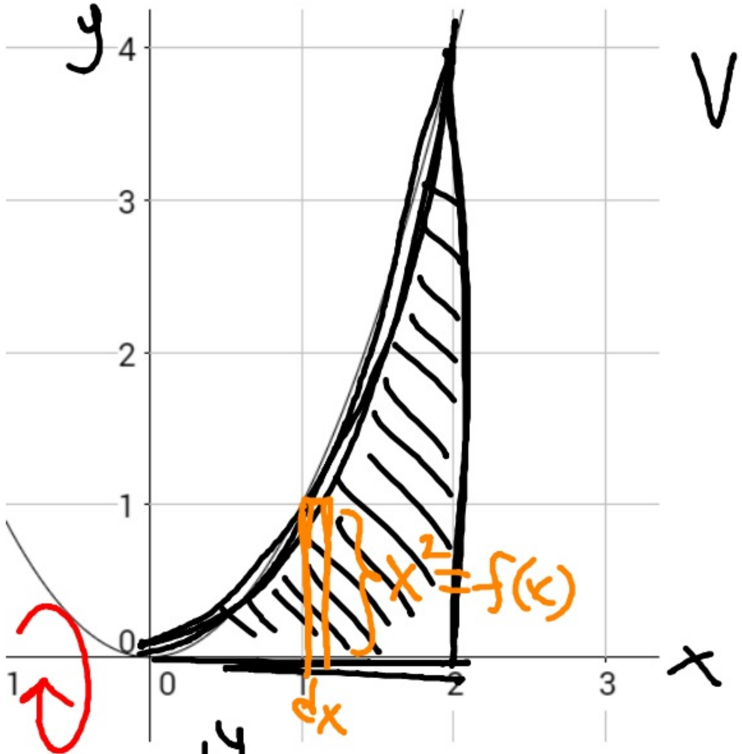


Volume of solid = sum of volume of slices





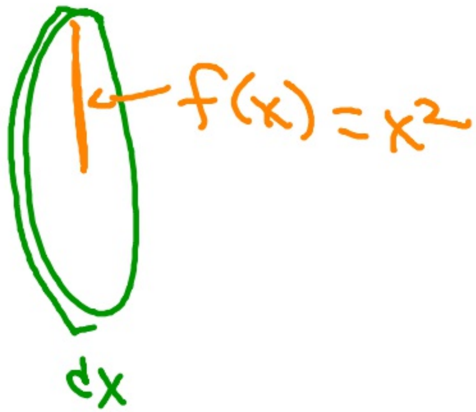
Concrete Example: Region bounded $y = x^2$, x-axis, and $x=2$ revolved around the x-axis.



$$V = \int_0^2 \pi \cdot (x^2)^2 \cdot dx = \pi \int_0^2 x^4 \, dx$$

$$\pi \left(\frac{1}{5} x^5 \right) \Big|_0^2$$

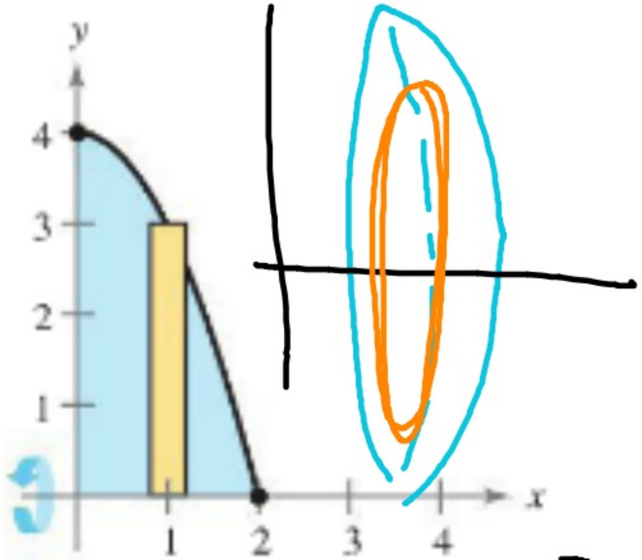
$$\frac{32}{5} \cdot \pi$$



Always sketch:

- revolution axis
- single rectangle
- single disk

2. $y = 4 - x^2$



$\pi \cdot r^2 \cdot dx$

$$V = \int_0^2 \pi (4-x^2)^2 \, dx$$

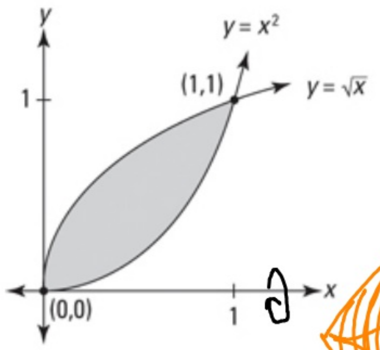
$$\pi \int_0^2 16 - 8x^2 - x^4 \, dx$$

$$\pi \cdot 16x - \frac{8}{3}x^3 - \frac{1}{5}x^5 \Big|_0^2$$

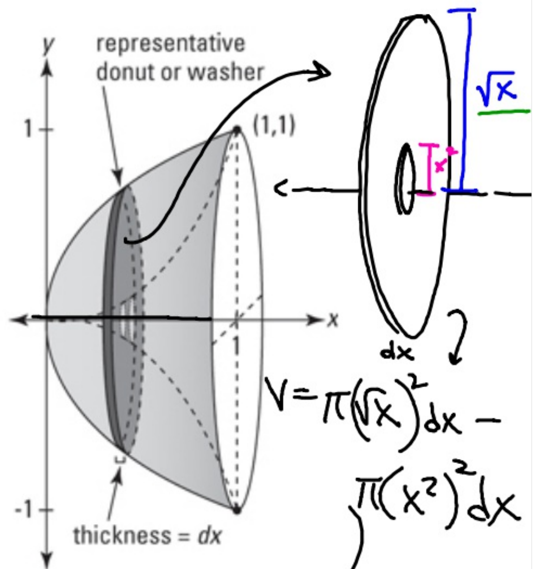
$$\pi\left(32 - \frac{8}{3} \cdot 8 - \frac{32}{5} - 0\right)$$

$$\boxed{\pi\left(32 - \frac{64}{3} - \frac{32}{5}\right)}$$

What about this?



now revolve this shaded area about the x-axis



$V=\pi(\sqrt{x})^2dx - \pi(x^2)^2dx$

$$\int_0^1 \pi(\sqrt{x})^2dx - \pi(x^2)^2dx$$

$$\pi\int_0^1\left[(\sqrt{x})^2-(x^2)^2\right]dx$$

$$\cancel{\int_a^b \pi(f(x)-g(x))^2dx}$$

Washer Method:

$$V=\int_a^b \pi\left(f(x)^2-g(x)^2\right)dx$$

where f is outer rim,
g is inner rim.
(relative to axis:)

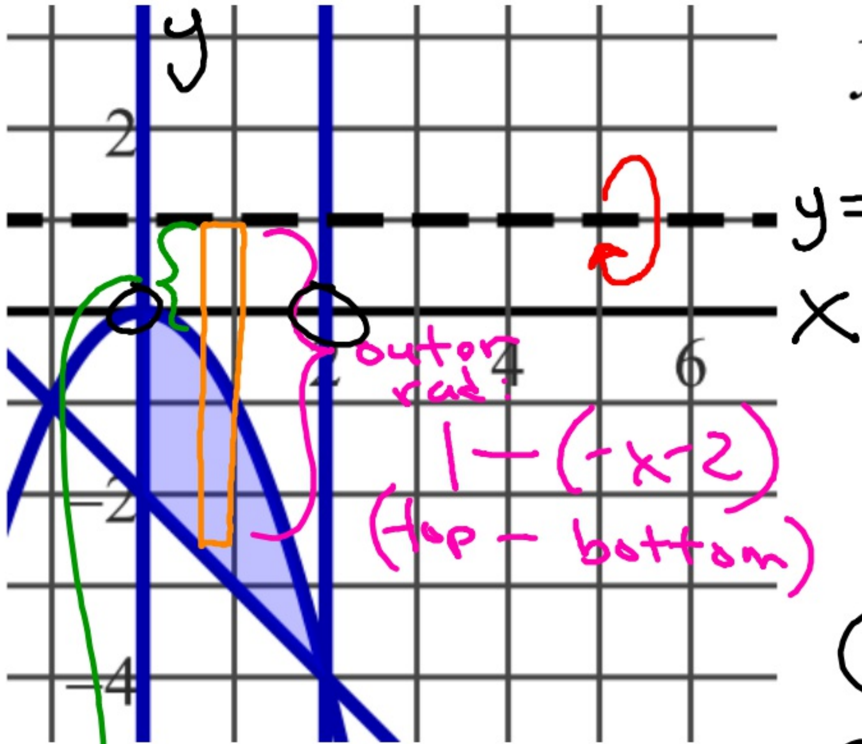
Disk method:

$$V = \pi \int_a^b (f(x))^2 \, dx$$

Washer method:

$$V = \pi \int_a^b (f(x)^2 - g(x)^2) \, dx$$

$$\underset{\text{top-bottom}}{\overset{f}{\text{outer}}} \text{ and } \underset{\text{top-bottom}}{\overset{g}{\text{inner}}} \text{ radius}$$ ***<u>RELATIVE</u>*** to axis of revolution



$$y=-x-2,\ \ y=-x^2$$

① Draw a rectangle attached to axis of rev.

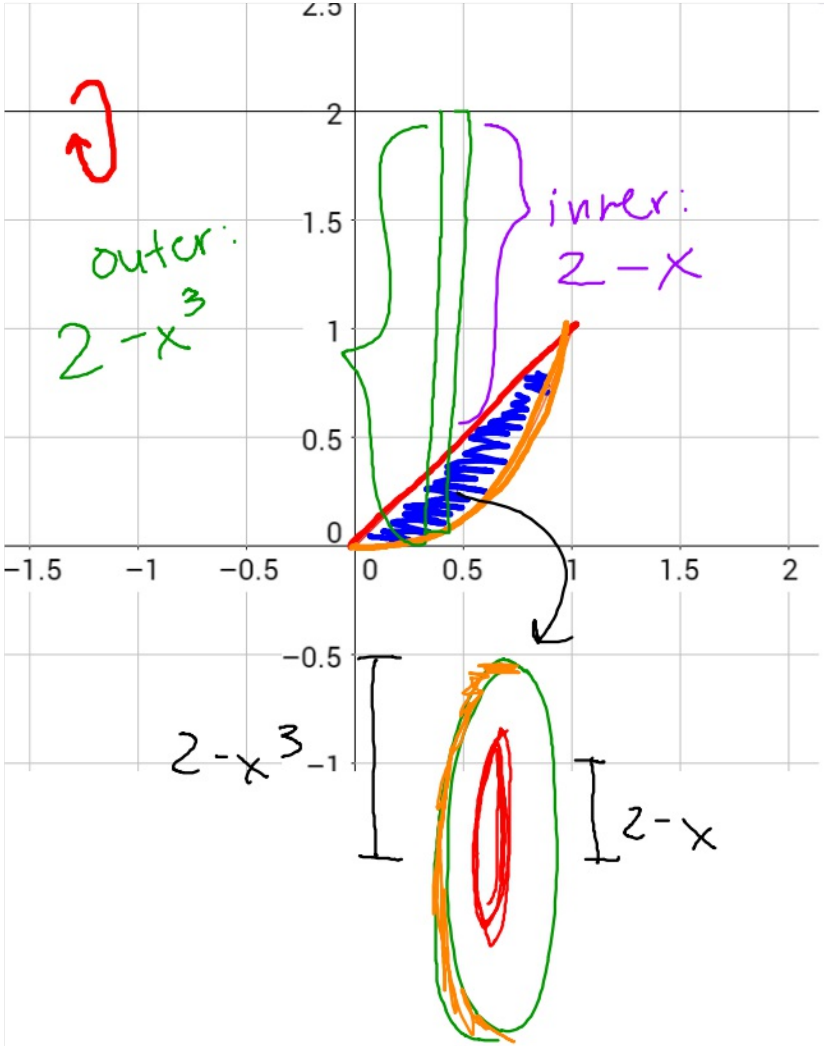
② carefully define expressions for outer/inner radii.

③ use formula

inner radius:
$1 - -x^2$
(top − bottom)

$$V=\int_0^2 \pi\left((x+3)^2-(1+x^2)^2\right)dx$$

☹



- $y=x$
- $y=x^3$

about $y=2$

$$V=\int_0^1 \pi\left(\left(2-x^3\right)^2-\left(2-x\right)^2\right)dx$$

math-9

handout, skip 3 and 4