Journal: 8/21

Evaluate the following limits using the given graph. If a limit d.n.e., explain using proper mathematical notation.

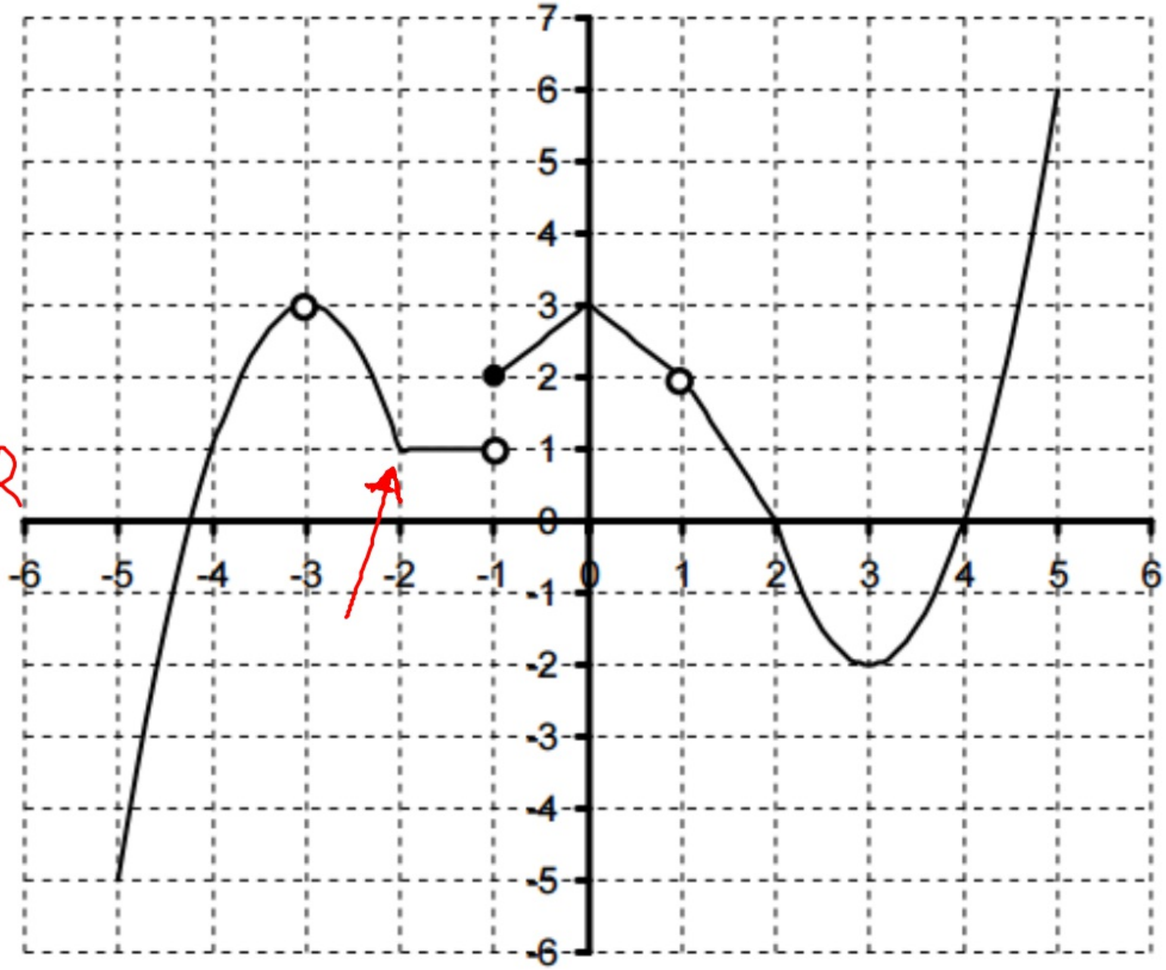
1 $\lim_{x \to -3} f(x) = 3$

$f(-3) = \text{undef.}$

2 $\lim_{x \to -1} f(x)$ dne

$\lim_{x \to -1^-} f(x) = 1 \neq \lim_{x \to -1^+} f(x) = 2$

3 $\lim_{x \to -2} f(x) = 1$



Homework Solutions

17. 2

18. 4

19. dne, -1≠ 1

20. dne, lim x->5- = -∞   lim x->5+ = ∞

21. dne, oscillates between 1 and -1

22. dne, left to infinity, right to negative infi

23.

a f(1) = 2

b dne, 3.5=! 1

c f(4) is undefined

d lim x->4 of f(x) = 2

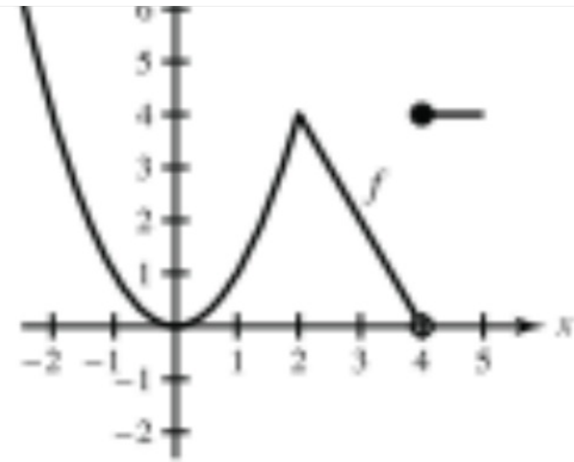
24.

es not exist. The vertical dotted line

hat f is not defined at –2.

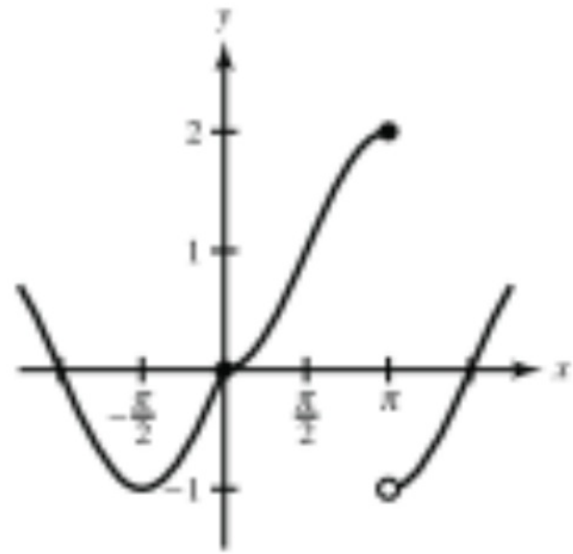
does not exist. As x approaches –2, the

$f(x)$ do not approach a specific number.

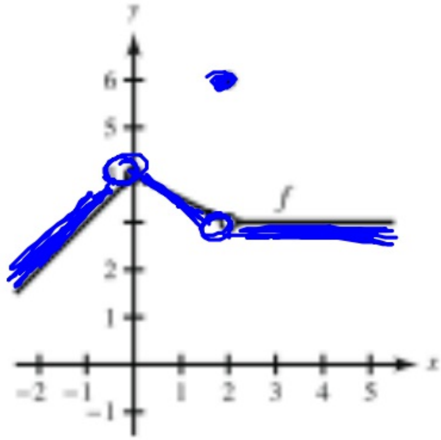


$\lim_{x\to c} f(x)$ exists for all values of

26.



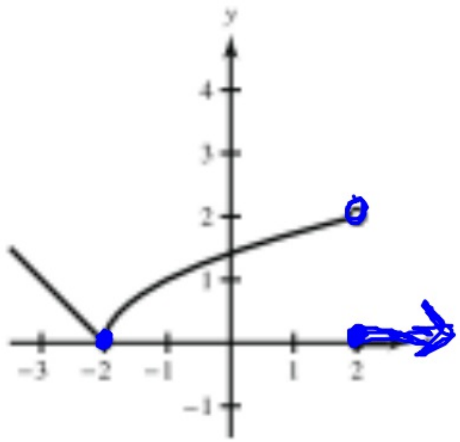
$\lim_{x\to c} f(x)$ exists for all values of e possible answer is

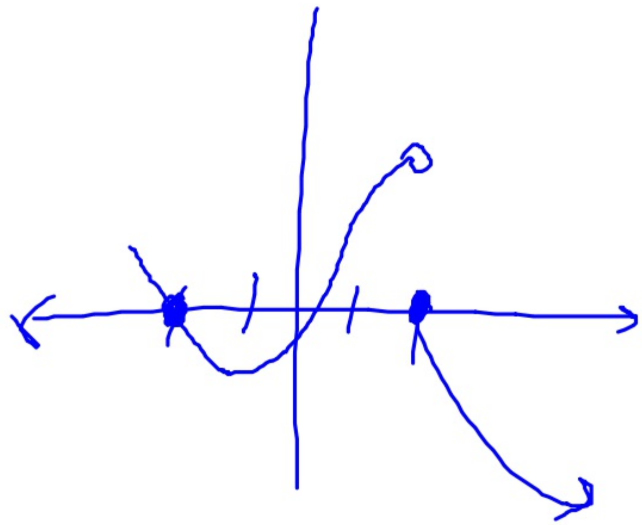


71. T

72. false. cannot approach from 0-

e possible answer is





. (a) $\lim_{x \to c} f(x)$ exists for all $c \neq -3$.

(b) $\lim_{x \to c} f(x)$ exists for all $c \neq -2, 0$.

Limits Algebraically

Well behaved functions

polynomials ex $x^4 + 3x^{12} + \dots$

trig functions

Direct Substitution

For a well-behaved function,

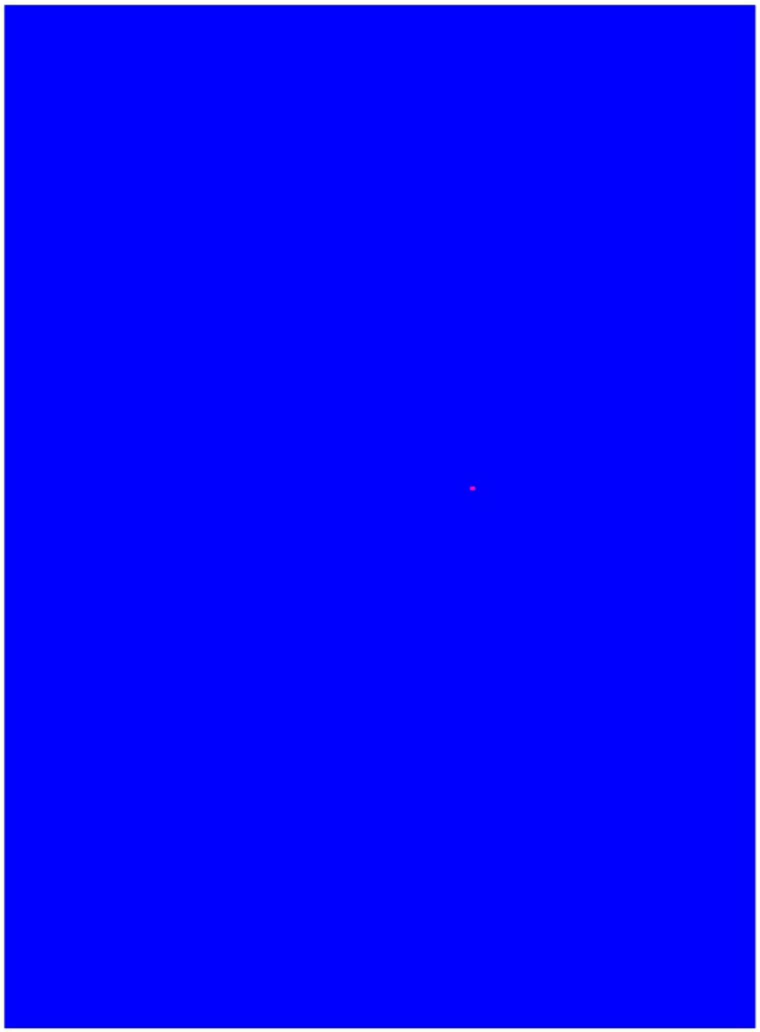
$$\lim_{x \to z} f(x) = f(z)$$

Properties of Limits: p 59

common sense

"factoring out" of a limit

limit of a product is...



Real #'s ~~ex~~

$$\lim_{x\to 5} 3\cdot\tan\left(\frac{\pi x}{3}\right)$$

$$3\cdot\lim_{x\to 5}\tan\left(\frac{\pi x}{3}\right)$$

$$3\cdot\tan\left(\frac{5\pi}{3}\right)$$

Ex

let $\lim_{x \to c} f(x) = A$ and $\lim_{x \to c} g(x) = B$

Find :

$\lim_{x \to c} 3g(x) = 3B$

$3 \cdot \boxed{\lim_{x \to c} g(x)}$ B

$\lim_{x \to c} f(x) * g(x) = AB$

$\lim_{x \to c} f \cdot \lim_{x \to c} g$

$\lim_{x \to c} f(x)\hat{}0.5 = \sqrt{A} = A^{1/2}$

Composite function

$\lim_{x \to c} \sin(g(x)) = \sin(B)$

Trig limits: p. 61

common sense for the most part

UNIT CIRCLE

Two Special Trig Limits

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

$$\lim_{x \to 0} \frac{1-\cos(x)}{x} = 0$$

lim sin(x)
x->0 x

lim 1-cos(x)
x->0 x

Using Special Trig Limits

ex

$$\lim_{x \to 0} \frac{\sin(3x)}{x}$$

$\cos(2x) \neq 2\cos(x)$

Weds.

$$\boxed{\lim_{x \to 0} \frac{\tan(x)}{x}}$$

Let $m = 3x \longrightarrow x = \frac{m}{3}$

as $x \to 0$, $m \to 0$

$$\lim_{m \to 0} \frac{\sin(m)}{\frac{1}{3}m}$$

$$\boxed{\tan(x) = \frac{\sin(x)}{\cos(x)}}$$

$$\lim_{m \to 0} 3 \cdot \frac{\sin(m)}{m}$$

$$3 \cdot \lim_{m \to 0} \frac{\sin(m)}{m} = 1 = 3$$

$$3 \cdot 1 =$$

Two Major Techniques for Evaluating Limits When Direct Sub. Fails

"massage"

Try direct sub with this:

lim x->2 $\frac{x^3 - 8}{x-2}$

$= \frac{2^3 - 8}{2-2} = \frac{0}{0}$!!

Looking for this...?

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$\frac{x^3 - 8}{x-2} = x^3 - 2^3$$

$$\lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$$

$$\lim_{x\to 2} x^2+2x+4 = 2^2+2\cdot 2+4$$

$$12$$

Factor, Cancel, Sub

Rationalization Technique

Try direct sub with this:

$$\lim_{x\to 3} \left(\frac{\sqrt{x+6} - 3}{x-3}\right) \cdot \underbrace{\left(\frac{\sqrt{x+6} + 3}{\sqrt{x+6} + 3}\right)}_{}$$

Conjugate

"1"

opp sign

$$\frac{x+6+3\cancel{\sqrt{x+6}}-3\cancel{\sqrt{x+6}}-9}{(x-3)(\sqrt{x+6}+3)}$$

$$\frac{x+6-9}{(x-3)(\text{——})} = \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+6}+3)}$$

$$\lim_{x\to 3} \frac{1}{\sqrt{x+6}+3} = \frac{1}{6}$$

Homework (due Weds.)

p. 67: 6-30 (multiples of 3), 40, 52-54, 65-67