SOLUTIONS MOHYUDDIN

F-L1c

Practice Assessment

Evaluate the following limits. Show all work.

1. $\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$ try direct sub.; $\frac{\sqrt{0+3}-\sqrt{3}}{0} = \frac{0}{0}$ indet. form

Rationalize:

conjugate

$$\lim_{x\to 0} \frac{(\sqrt{x+3}-\sqrt{3})}{x} \cdot \frac{(\sqrt{x+3}+\sqrt{3})}{\sqrt{x+3}+\sqrt{3}} \xrightarrow{\text{FOIL}} \lim_{x\to 0} \frac{x+\cancel{3}-\cancel{3}}{x(\sqrt{x+3}+\sqrt{3})} \Rightarrow \lim_{x\to 0} \frac{\cancel{x}\ 1}{\cancel{x}(\sqrt{x+3}+\sqrt{3})}$$

$$\lim_{x\to 0} \frac{1}{\sqrt{x+3}+\sqrt{3}} \to \frac{1}{\sqrt{3}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

2. $\lim_{x\to 0} \frac{\sin(3x)}{2x}$ Direct sub... $\frac{\sin(0)}{0} = \frac{0}{0}$: indet. form

Note typo

MUST KNOW

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$

"1"

$$\lim_{x\to 0} \frac{\sin 3x}{2x} \cdot \frac{3}{3} \Rightarrow \lim_{x\to 0} \left[\frac{3\cdot \sin 3x}{2\cdot 3x}\right] \Rightarrow \lim_{x\to 0} \left(\frac{3}{2} \cdot \frac{\sin 3x}{3x}\right)$$

"limit of products is product of limits"

$$\lim_{x\to 0} \frac{3}{2} \cdot \boxed{\lim_{x\to 0} \frac{\sin 3x}{3x}}$$ yay!

$$\Downarrow$$

$$\frac{3}{2} \cdot 1 = \frac{3}{2}$$

F-L1b

Evaluate the following limits. Show all work.

3. $\lim_{x\to -7^-} -\frac{x}{x+7}$

$$-\frac{-7^-}{-7^-+7} \Rightarrow -\frac{-7^-}{0^-}$$

-7^- ← negative #

0^- ← negative #

$-\frac{-7.00001}{-0.0000\ldots1}$

$-7.00001+7$

$$\Downarrow$$

$$-+\infty$$

$$\downarrow$$

$$-\infty$$

4. $\lim_{x\to\infty} \frac{-2x^5+8x^4-7x^3+45x-0.178}{4x^5-3x^3+25x^2-0.27x+1224}$

Replace x with infinity. But realize: $\infty^5 >>>>> \infty^4 >>>> \infty^3$, etc...

$$\frac{-2\cdot\infty^5 \boxed{+8\infty^4 + \ldots \text{etc.}}}{4\infty^5 \boxed{-3\infty^3 + \ldots \text{etc}}}$$

← All this is jack squat compared to $-2\cdot\infty^5$

← All this is measly crumbs compared to $4\cdot\infty^5$

$$\frac{-2\,\cancel{\infty^5}}{4\,\cancel{\infty^5}}$$ "equally huge"

$$\Rightarrow \frac{-2}{4} \to -\frac{1}{2}$$

F-B1

5. Identify any vertical asymptote(s) of the following function. Justify your answer using limits.
$f(x) = \frac{x+1}{x^2-1}$

$f(x) = \frac{\cancel{x+1}}{\cancel{(x+1)}(x-1)} \Rightarrow f(x) = \frac{1}{x-1}$

V. A. candidate: looks like $x = 1$

Justify ?

$\lim_{x \to 1^+} \frac{1}{x-1} \Rightarrow \frac{1}{1^+ - 1} = \frac{1}{1.000...1 - 1} = \frac{1}{0^+} = +\infty$

Check to see if either 1^+ and/or 1^- produces $\pm\infty$

[see V.a. definition, 8/26 notes]

So, $x = 1$ is a V.a.

6. Identify any horizontal asymptote(s) of the following function. Justify your answer using limits.
$f(x) = \frac{x+1}{x^2-1}$

def. of H.A. Notes: 8/26

$y = b$ is H.A. $\Leftrightarrow \lim_{x \to \infty} f(x) = b$, $b \in \mathbb{R}$

$\lim_{x \to \infty} \frac{x+1}{x^2-1} \rightarrow \frac{\infty + 1}{\infty^2 - 1}$ chomp change compared to infinity.

$\frac{\infty}{\infty^2}$ ← big / ← freaking GINORMOUS

$= 0$

So $y = 0$ is a H.A.