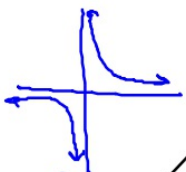
Limits Yielding Infinity and Limits at infinity

Consider $f(x) = \dfrac{1}{x}$

What is $f(0)$? = undefined.

What is $\lim\limits_{x\to 0} f(x)$ = dne.



$\lim\limits_{x\to 0^-} f(x) = -\infty \neq \lim\limits_{x\to 0^+} f(x) = \infty$

x	$f(x)$
-0.001	-1000

x	$f(x)$
.001	1000
.00002	5000
.000001	100,000

$\frac{1}{0} \approx \infty$

$$f(x) = \frac{1}{x^2}$$

$$\lim_{x \to 0} f(x)$$

$$\lim_{x \to 0^-} f(x) = \frac{1}{(0^-)^2} = \frac{1}{0^+} = \infty$$

"really small neg. #"

$$\lim_{x \to 0^+} f(x) = \frac{1}{(0^+)^2} = \frac{1}{0^+} = \infty$$

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$

$$f(x) = \frac{-1}{(x-4)^2}$$

$$\lim_{x \to 4} \frac{-1}{(x-4)^2}$$

$$\lim_{x \to 4^-} \frac{-1}{(x-4)^2} = \frac{-1}{(4^- - 4)^2} = \frac{-1}{(0^-)^2}$$

"3.999"

$$\frac{-1}{0^+} = \underline{-\infty}$$

$$\lim_{x \to 4^+} f(x) = \frac{-1}{(4^+ - 4)^2}$$

$$\frac{-1}{(0^+)^2}$$

$$\frac{-1}{0^+} = -\infty$$

Vertical Asymptotes as Limits Yielding Infinity

The function f(x) will have a vertical asymptote at x=a (equation of a vertical line) if any of the following are true:

$$\lim_{x\to a^-} f(x) = \pm\infty \qquad \lim_{x\to a^+} f(x) = \pm\infty \qquad \lim_{x\to a} f(x) = \pm\infty$$

VA if a limit at a specific value produces infinity

Horizontal Asymptotes as Limits at Infinity

A function f(x) will have a horizontal asymptote at y=L if either is true:

$$\lim_{x\to\infty} f(x) = L \qquad\qquad \lim_{x\to-\infty} f(x) = L$$

"H.A.?"

HA if a limit at infinity produces a specific value

"Rules" from Precal for Horizontal Asymptotes

Let h(x) = $\dfrac{f(x)}{g(x)}$

If F and G are both polynomial functions, then:

- If F's degree is bigger, then there is no HA because $\lim_{x \to \pm\infty} h(x) = \pm\infty$

ex $\displaystyle \lim_{x \to \infty} \frac{2x^{10} - 4x^4 + 2}{3x^2} = \infty$

- If G's degree is bigger, then the HA is y=0 because $\lim_{x \to \pm\infty} h(x) = 0$

ex $\displaystyle \lim_{x \to \infty} \frac{2x^2 + 4x - 3}{-x^3 + 4x - 5000} = 0$

- If F and G have the same degree, then the HA is the ratio of the leading coeff.

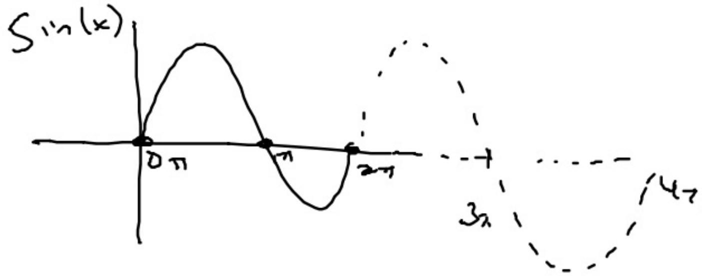
ex: $\displaystyle \lim_{x \to \infty} \frac{4x^4 \cancel{+ 3x^2} \cancel{- 2x} \cancel{+ 1}}{3x^4 \cancel{+ 5x^3} \cancel{- x} \cancel{+ 3}} = \frac{4}{3}$

If the functions are not polynomial functions, consider their dominance.
If the dominating function is on top, no HA.
If the dominating function is on bottom, HA is y=0
If the functions are equally dominating, the HA is a ratio of their bases/coeff./etc

Homework: close reading of §1.4 (p70-78)

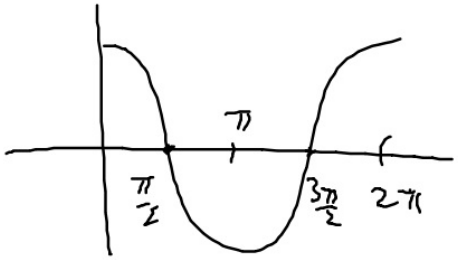
25)

$f: \csc(\pi x) = \dfrac{1}{\sin(\pi x)}$ $\quad \sin(\pi x) = 0$ when x is a whole #.



$\sin(\pi x)$

$\cos(\pi x) = 0$



Let $\theta = \pi x$

$\cos(\theta) = 0$

$\theta = \frac{\pi}{2} + 2\pi n$ $\qquad$ $\theta = \frac{3\pi}{2} + 2\pi n$

$\cancel{\pi} x = \frac{\cancel{\pi}}{2} + 2\cancel{\pi} n$ $\qquad$ $\cancel{\pi} x = \frac{3\cancel{\pi}}{2} + 2\cancel{\pi} n$

$x = \frac{n}{2}$ where n is odd $\qquad$ $\cancel{x} = \frac{1}{2} + 2n$ $\qquad$ $x = \frac{3}{2} + 2n$