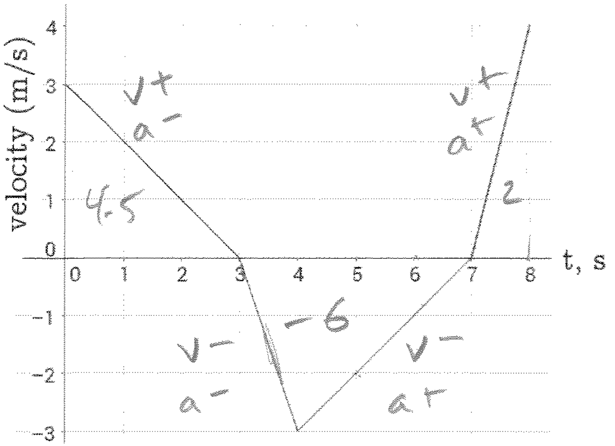
Particle Motion: A Graphical Approach

$s(0) = 8$

Here is the graph of the velocity of a particle moving along the x-axis. At time t=0, its position s(0) is 8 m.



Include units in each response:

1. Find the velocity ~~of the~~ particle when t=6.

$v(6) = \boxed{-1 \text{ m/s}}$

2. Find the speed of the particle when t=5.

$|v(t)|$ ✓

$|v(5)| = |-2| = \boxed{2 \text{ m/s}}$

3. Find the acceleration when t=2.

$a(t) = v'(t)$ slope @ 2

$\boxed{-1 \text{ m/s/s or m/s}^2}$

4. Find the position at t=7 [So, find s(7)]

$s(7) = s(0) + \int_0^7 v(t)\,dt$

$= 8 + (4.5 - 6) = 8 + (-1.5)$

$= 6.5 \text{ m}$

5. If velocity is v(t) then speed is.... $|v(t)|$

6. Total distance traveled over 8 seconds: ✓

$\int_0^8 |v(t)|\,dt = 4.5 + 6 + 2 = \boxed{12.5 \text{ m}}$

7. Write an expression to represent the average speed over these 8 seconds. Then, find this average speed.

Avg. value of speed function.

$\frac{1}{8-0}\int_0^8 |v(t)|\,dt \rightarrow \frac{1}{8}(12.5) \rightarrow \boxed{1.5625 \text{ m/s}}$

#6

8. When is the particle at rest? $\boxed{t = 3, 7}$
9. When does the particle change direction? $\boxed{t = 3, 7.}$ v changes sign.

10. Over what interval(s) is the speed decreasing?

v(t) and a(t) diff. signs $\rightarrow \boxed{(0, 3) \text{ and } (4, 7)}$

11. Over what interval(s) does the object move left? v(t) negative. $\boxed{(3, 7)}$

12. When is the particle farthest to the right? What is the position at this time? $t = 3$, $s(3) = 12.5$m

visual.

(t=8) +2

(t=7) −6

+4.5 (t=3)

8 (t=0)

Absolute max of position

rel max? → t = 3. $s(3) = s(0) + \int_0^3 v(t)\,dt$

$8 + 4.5 = 12.5$

Endpoints → $s(0) = 8$

$s(8) = s(0) + \int_0^8 v(t) = 8 + (4.5 - 6 + 2)$

$8 + (0.5)$

8.5

13. Is the particle accelerating or decelerating at t=3.5? Justify.

Accelerating v(3.5) negative } same sign.
a(3.5) negative

14. When does the particle move the fastest?

max of speed. t = 8. $(|v(8)| = 4 \text{ m/s})$

Particle Motion: An Analytical Approach [Inspired by 2003AB4 no calculator]

A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$ feet per second.

1. Find the speed of the particle when t = 3 seconds.

$|v(t)|$ $\quad |v(3)| = |-1+e^{1-3}| = |\underbrace{-1+e^{-2}}_{\text{negative}}| = -(-1+e^{-2}) \rightarrow \boxed{1-e^{-2} \text{ ft/sec}}$

2. Find the acceleration of the particle when t=3 seconds?

$a(t) = v'(t) = -e^{1-t}$; $a(3) = -e^{1-3} = \boxed{-e^{-2} \text{ ft/s}^2}$

3. Is the speed of the particle increasing or decreasing when t =3 seconds? Justify.

v(3) is negative a(3) is negative so, Speed is [✓ increasing.] (v(3) and a(3) have same sign)

4. Find all times where the particle changes direction. vel. is 0, changes sign.

$v(t) = -1 + e^{1-t} = 0$

$e^{1-t} = 1$ $\rightarrow$ $\ln(e^{1-t}) = \ln(1)$

$1 - t = 0$ $\rightarrow$ $\underline{t=1}$ $\quad \overset{v}{t} \longleftarrow + \underset{1}{|} - \longrightarrow$ $\quad$ t = 1 sec

5. When t=1 sec, the particle is located at 3 feet [s(1)=3] Find the position of the particle at t=5 sec.

$s(5) = s(1) + \int_1^5 v(t)\,dt$ $\rightarrow$ $\int_1^5 -1 + e^{1-t}\,dt \Rightarrow [-t - e^{1-t}]_1^5$

$3 + \int_1^5 v(t)\,dt$ $\quad$ $[(-5 - e^{-4}) - (-1 - \overset{1}{\cancel{e^{0}}})]$ $\quad (-2)$

$3 + (-3 - e^{-4}) \Rightarrow$ $-e^{-4}$ ft $\quad$ $[-5 - e^{-4} + 2] \Rightarrow \underline{-3 - e^{-4}}$

6. Find the total distance traveled over the first 3 seconds.

$(0, 3)$

$\int_0^3 |v(t)|\,dt$ $\quad$ Note: $\overset{v}{t} \longleftarrow + \underset{1}{|} - \longrightarrow$

$\int_0^1 \overset{\text{pos}}{|v(t)|}\,dt + \int_1^3 \overset{\text{neg}}{|v(t)|}\,dt$

$\int_0^1 -1 + e^{1-t}\,dt + \int_1^3 -(-1 + e^{1-t})\,dt$ $\quad$ $1 - e^{1-t}$

$\underbrace{[-t - e^{1-t}]_0^1}_{A} + \underbrace{[t + e^{1-t}]_1^3}_{B}$

A: $[(-1 - \overset{1}{\cancel{e^{1-1}}}) - (-0 - e^1)]$

$[-2 - -e] \rightarrow -2 + e$

B: $[(3 + e^{-2}) - (1 + \underset{2}{\cancel{e^{0}}})]$

$[3 + e^{-2} - 2] \Rightarrow 1 + e^{-2}$

$\underbrace{-2 + e}_{A} + \underbrace{1 + e^{-2}}_{B} \Rightarrow \boxed{-1 + e + e^{-2} \text{ ft}}$

Particle Motion: Numerically [2015AB1, no calculator]

t minutes	0	12	20	24	40
$v(t)$ m/min	0	200	240	-220	150

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\,dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)|\,dt$ using a right Riemann sum with the four subintervals indicated in the table.

Particle Motion: A Numerical Approach [2015AB1, no calculator]

12 8 4 16

t minutes	0	12	20	24	40
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RRAM

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

$$v'(16) \approx \frac{240-200}{20-12} = \frac{40}{8} = 5 \text{ m/mi}$$

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\,dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)|\,dt$ using a right Riemann sum with the four subintervals indicated in the table.

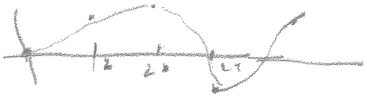
Distance traveled in meters over 40 minutes.

$$\int_0^{40} |v(t)|\,dt \approx 12(200) + 8(240) + 4(220) + 16(150)$$

$$2400 + 1920 + \overset{\text{abs. vel.}}{880} + 2400$$

$$= 7600 \text{ meters.}$$

(c) At least how many times in (0,40) was Johanna at rest? Explain.



2 times. I.V.T.

(d) Find Johanna's position after 24 minutes if her initial position s(0) is 3 feet from a landmark. RRAM.

$$s(24) = s(0) + \int_0^{24} v(t)\,dt$$

$$3 + [12(200) + 8(240) + 4(-220)]$$

$$3 + [2400 + 1920 - 880]$$

$$3 + (3440) = \boxed{3443 \text{ m}}$$

(e) Find Johanna's average speed over the 40 minute interval.

Avg. Value of Speed.

$$\frac{1}{40-0}\underbrace{\int_0^{40} |v(t)|\,dt}_{\text{see b.}} \Rightarrow \frac{1}{40}(7600) = \frac{760}{4} \text{ m/min}$$

$$= 190 \text{ m/min}$$