## Particle Motion: A Graphical Approach

Here is the graph of the velocity of a particle moving along the x -axis. At time $\mathrm{t}=0$, its position $\mathrm{s}(0)$ is 8 m .
Include units in each response:


1 . Find the velocity of the particle when $t=6$.
2. Find the speed of the particle when $\mathrm{t}=5$.

3 . Find the acceleration when $t=2$.
4. Find the position at $\mathrm{t}=7$ [So, find $\mathrm{s}(7)]$
5. If velocity is $\mathrm{v}(\mathrm{t})$ then speed is....
6. Total distance traveled over 8 seconds:
7. Write an expression to represent the average speed over these 8 seconds. Then, find this average speed.
8. When is the particle at rest?
9. When does the particle change direction?
10. Over what interval(s) is the speed decreasing?
11. Over what interval(s) does the object move left?
12. When is the particle farthest to the right? What is the position at this time?
13. Is the particle accelerating or decelerating at $\mathrm{t}=3.5$ ? Justify.
14. When does the particle move the fastest?

Particle Motion: An Analytical Approach [Inspired by 2003AB4 no calculator]
A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t)=-1+e^{1-t}$ feet per second.

1. Find the speed of the particle when $t=3$ seconds.
2. Find the acceleration of the particle when $t=3$ seconds.
3. Is the speed of the particle increasing or decreasing when $\mathrm{t}=3$ seconds? Justify.
4. Find all times where the particle changes direction.
5. When $\mathrm{t}=1 \mathrm{sec}$, the particle is located at 3 feet $[\mathrm{s}(1)=3]$ Find the position of the particle at $\mathrm{t}=5 \mathrm{sec}$.
6. Find the total distance traveled over the first 3 seconds.

| $t$ minutes | 0 | 12 | 20 | 24 | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $v(t) \mathrm{m} / \mathrm{min}$ | 0 | 200 | 240 | -220 | 150 |

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function $v$. Selected values of $v(t)$, where $t$ is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.
(a) Use the data in the table to estimate the value of $v^{\prime}(16)$.
(b) Using correct units, explain the meaning of the definite integral $\int_{0}^{40}|v(t)| d t$ in the context of the problem.

Approximate the value of $\int_{0}^{40}|v(t)| d t$ using a right Riemann sum with the four subintervals indicated in the table.
(c) At least how many times in $(0,40)$ was Johanna at rest? Explain.
(d) Find Johanna's position after 24 minutes if her initial position $s(0)$ is 3 feet from a landmark.
(e) Find Johanna's average speed over the 40 minute interval.

A particle moves along the $x$-axis so that its velocity at time $t$, for $0 \leq t \leq 6$, is given by a differentiable function $v$ whose graph is shown above. The velocity is 0 at $t=0, t=3$, and $t=5$, and the graph has horizontal tangents at $t=1$ and $t=4$. The areas of the regions bounded by the $t$-axis and the graph of $v$ on the intervals $[0,3],[3,5]$, and $[5,6]$ are 8,3 , and 2 , respectively. At time $t=0$, the particle is at $x=-2$.
(a) For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
(b) For how many values of $t$, where $0 \leq t \leq 6$, is the particle at $x=-8$ ? Explain your reasoning.
(c) On the interval $2<t<3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.


Graph of $v$

Analytical Practice: 2011AB1 calculator active

For $0 \leq t \leq 6$, a particle is moving along the $x$-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t)=2 \sin \left(e^{t / 4}\right)+1$. The acceleration of the particle is given by $a(t)=\frac{1}{2} e^{t / 4} \cos \left(e^{t / 4}\right)$ and $x(0)=2$.
(a) Is the speed of the particle increasing or decreasing at time $t=5.5$ ? Give a reason for your answer.
(b) Find the average velocity of the particle for the time period $0 \leq t \leq 6$.
(c) Find the total distance traveled by the particle from time $t=0$ to $t=6$.
(d) For $0 \leq t \leq 6$, the particle changes direction exactly once. Find the position of the particle at that time.

