

Good afternoon:

assessment is rescheduled to Monday due to short class period

Check your calendars: switch Apr 20 and Apr 23

When bell rings, have notebooks out as we have a lot to cover in a short period of time. Finale seniors dismissed at 2:20p and not a minute before

Upcoming dates:

- M 4/23 assessment
- F 4/27 take home assessment due (getting it today)
- F 5/4 roller coasters due; 2016 AP mult choice packet due



Constant air volume

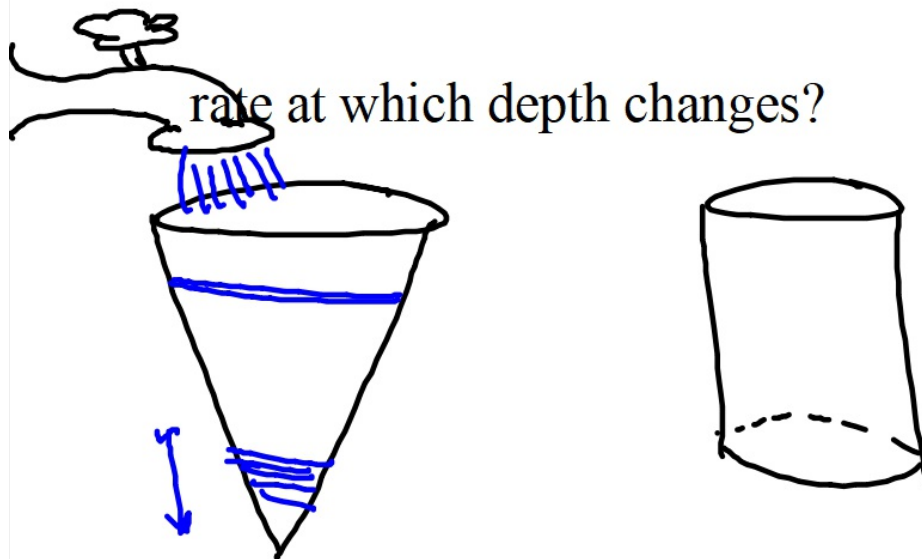
(no, but the rates  
are related to each other)

Is the surface area growing at a constant rate?!

Water pouring into a cup

constant rate of water leaving faucet

related, but not equal



## Understanding "derivative with respect to"

Suppose  $x$  and  $y$  are functions of time  $t$ . Find  $dy/dt$

$$\text{where } \frac{d}{dt}(3x^2 + 5y^2) = 12 \frac{d}{dt}$$

$$6x \frac{dx}{dt} + 10y \frac{dy}{dt} = 0$$

$$10y \frac{dy}{dt} = -6x \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-6x \cdot \frac{dx}{dt}}{10y}$$

Implicit differentiation, doing  $d/dt$  to both sides instead of  $d/dx$



slippery mess. A gust of wind of constant velocity nudges the ladder so that the base of the ladder slides away from the wall at a rate of 4 feet per second. How fast is the top of the ladder sliding down the wall when the base of the ladder is 8 feet from the wall?

### The Falling Ladder Problem

\* S D M P D  
 diagram  
 rate  
 equation  
 derivative ( $\frac{d}{dt}$ )  
 substitute,  
 solve

$\frac{dx}{dt} = 4 \text{ ft/sec}$

$x^2 + y^2 = 10^2$

$\frac{d}{dt} (x^2 + y^2) = (100) \frac{d}{dt}$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$8^2 + y^2 = 10^2$   
 $y = 6$

$2(8) \cdot 4 + 2(6) \cdot \frac{dy}{dt} = 0$

$64 + 12 \frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{64}{12} \rightarrow -5.333 \frac{\text{ft}}{\text{s}}$

Everything in Eq. except unknown should be known

$\frac{dy}{dt} = ?$

D Diagram

R Rates (label given information, needed information)

E Equation (geometry formula, usually. Be sure needed info is only unknown)

D Differentiate (with respect to TIME)

S Substitute, then solve



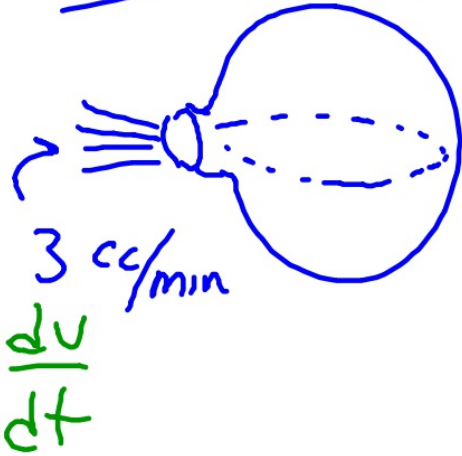
(you should write these down)

A spherical balloon is being inflated by a pump at a rate of 3 cubic centimeters per minute. At what rate is the radius of the balloon changing when the diameter of the balloon is 4 cm?

DREDS

$$\frac{dr}{dt} = ?$$

$$D=4 \Rightarrow \underline{\underline{r=2}}$$



$$\frac{d}{dt} \left( V = \frac{4}{3} \pi r^3 \right) dt$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$3 = 4\pi \cdot 2^2 \cdot \frac{dr}{dt}$$

$$3 = 16\pi \cdot \frac{dr}{dt}$$

$$\frac{3}{16\pi} = \frac{dr}{dt}$$

cm/min

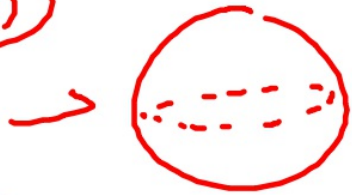
← Stopped here  
in class





A spherical snowball melts such that its radius decreases at a rate of 2 in/min. At what rate is the volume of the snowball changing when the radius is 3 inches?

(D)



(E)  $\frac{dV}{dt} = ?$

$\frac{dr}{dt} = -2$   $r = 3$

(F)

$$V = \frac{4}{3} \pi r^3$$

(P)

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

Red arrows point from the question mark under  $\frac{dV}{dt}$  and the checkmarks under  $r^2$  and  $\frac{dr}{dt}$ .

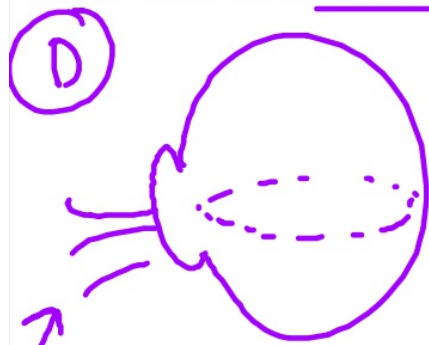
(S)

$$\frac{dV}{dt} = 4\pi \cdot 3^2 \cdot -2$$

$$= -72\pi \text{ in}^3/\text{min}$$

A <sup>Sph.</sup> balloon is being inflated such that its radius grows at a rate of 2 in per minute.

How fast is the surface area changing when the balloon's volume is  $32/3 \pi \text{ in}^3$ ?



$$\frac{dS}{dt} = ?$$

$$\frac{dr}{dt} = 2 \text{ in/min}$$
$$V = \frac{32}{3} \pi$$

Sph. Vol

Need this!

$$V = \frac{32}{3} \pi = \frac{4}{3} \pi r^3$$

(R)

$$S = 4\pi r^2$$

(D)

$$\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

?

$$32\pi = 4\pi r^3$$

$$32 = 4r^3$$

$$8 = r^3$$

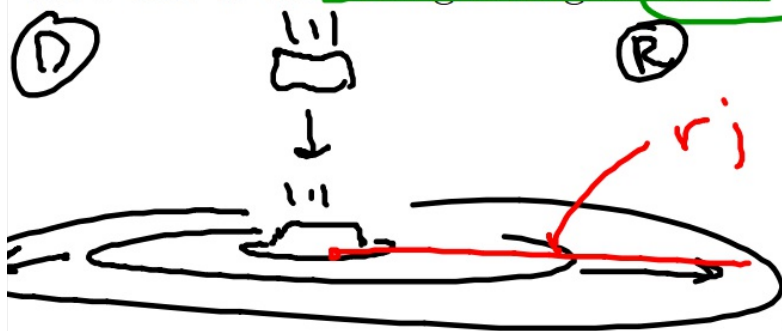
$$2 = r$$

(S)

$$\frac{dS}{dt} = 8\pi \cdot 2 \cdot 2$$

$$= 32\pi \text{ in}^2/\text{min}$$

A ripple forms in a pond. It is expanding such that its radius grows by 2 inches per second.  
 How fast is the area growing when the ripple's circumference is  $32\pi$  in?



(R)  $\frac{dr}{dt} = 2$        $\frac{dA}{dt} = ?$

$C = 32\pi$   
 $C = 2\pi r$

(E)  $A = \pi r^2$

$\frac{dA}{dt} = \underbrace{2\pi r}_{\text{given!}} \cdot \underbrace{\frac{dr}{dt}}_{\checkmark}$

(S)  $\frac{dA}{dt} = 32\pi \cdot 2$   
 $= 64\pi \text{ in}^2/\text{s}$

