Good afternoon:
no warm up ; have your related rates notes out from Friday

Reminders:
take home test due Friday

A spherical snowball melts such that its radius decreases at a rate of 2 in/min. At what rate is the volume of the snowball changing when the radius is 3 inches?

$V = \frac{4}{3}\pi r^3$

DREDS

R

$\frac{dV}{dt} = ?$ @ $r = 3$

D

$\frac{dr}{dt} = -2$

E

$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)\frac{d}{dt}$

D

$\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3r^2 \cdot \frac{dr}{dt}$

S

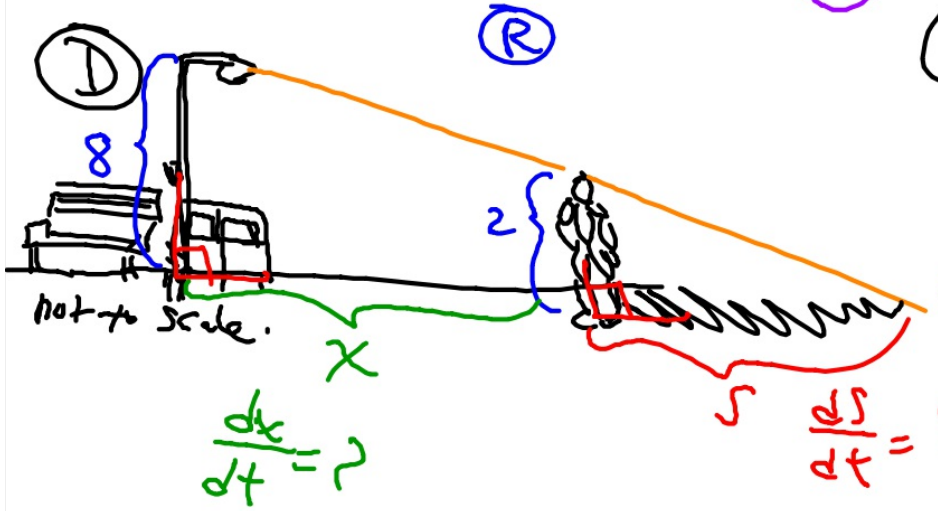
$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi \cdot 9 \cdot -2$

$\frac{dV}{dt} = -72\pi \text{ in}^3/\text{min}$

A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?

(A) $\frac{4}{27}$ (B) $\frac{4}{9}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$ (E) $\frac{16}{9}$



E

$\frac{s}{2} = \frac{x+s}{8}$

$8s = 2x + 2s$

D S

$6s = 2x$

$\frac{d}{dt}(3s) = (x)\frac{d}{dt}$

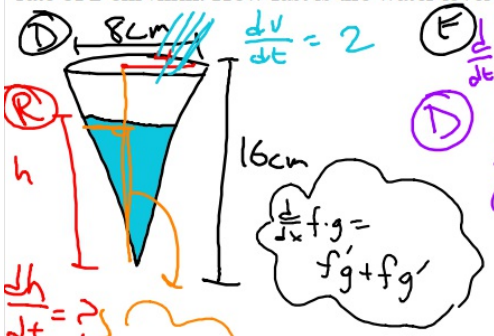
$3\frac{ds}{dt} = \frac{dx}{dt}$

$3 \cdot \frac{4}{9} = \frac{dx}{dt}$

$\frac{12}{9} = \frac{dx}{dt}$

$\frac{4}{3}$

Water is being poured into a conical cup with radius 4cm and height 16cm at a constant rate of 2 cm^3/min. How fast is the water level rising when the water is 10cm deep?



D $\frac{dV}{dt} = 2$

E $\frac{d}{dt}\left(V_c = \frac{1}{3}\pi r^2 \cdot h\right)\frac{d}{dt}$

D $\frac{dV}{dt} = \frac{\pi}{3}\left(2r \cdot \frac{dr}{dt} h + r^2 \cdot \frac{dh}{dt}\right)$ ☹ ☹ ☺ ☹ ?

$\frac{d}{dx} f \cdot g = f'g + fg'$

dead end!
no r! no dr/dt!

$\frac{dh}{dt} = ?$

$h = 4r$

$\frac{h}{4} = r$

E $V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 \cdot h$

$V = \frac{1}{3}\pi \cdot \frac{h^2}{16} \cdot h$

D $V = \frac{\pi}{48} h^3$

$\frac{dV}{dt} = \frac{\pi}{48} \cdot 3h^2 \cdot \frac{dh}{dt}$ ☺ ☺ ?

$\frac{dh^3}{dt} = \frac{d}{dh}h^3 \cdot \frac{dh}{dt}$

$\frac{8}{25\pi}$

$24 \cdot \frac{}{75\pi}$

S $2 = \frac{\pi}{48} \cdot 300 \cdot \frac{dh}{dt}$

$\frac{96}{300\pi} = \frac{dh}{dt}$ ← $\frac{2}{\frac{300\pi}{48}} = \frac{\frac{300\pi}{48} \cdot \frac{dh}{dt}}{\frac{300\pi}{48}}$

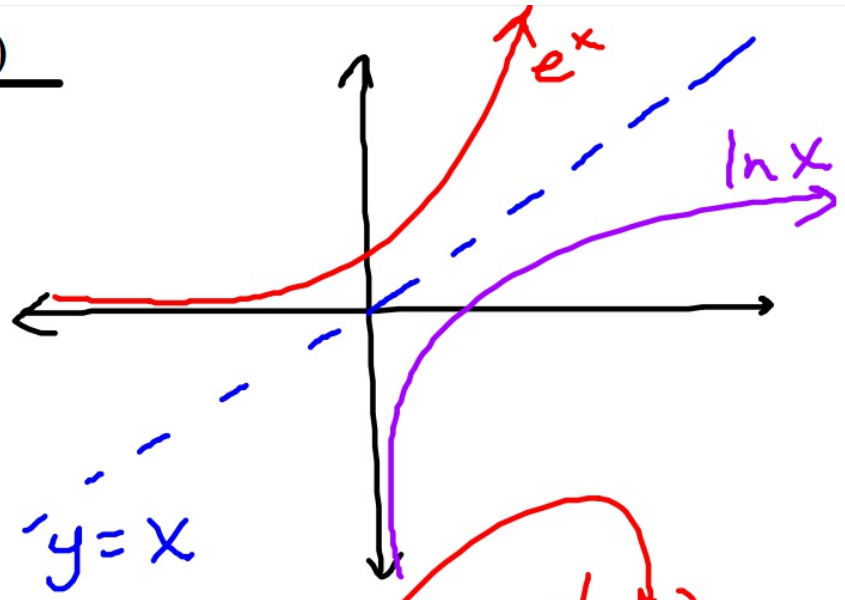
554 (AP, 1970AB). A right circular cone and a hemisphere have the same base, and the cone is inscribed in the hemisphere. The figure is expanding in such a way that the combined surface area of the hemisphere and its base is increasing at a constant rate of 18 square inches per second. At what rate is the volume of the cone changing at the instant when the radius of the common base is 4 inches?

Bonus Challenge!

Derivatives of Inverses (our last new topic 😭)

What is an inverse function?

y^{-1} f^{-1}



Ex) Find the inverse, f^{-1}.

$f(x) = 2x - 6$

$y = 2x - 6$

$x = 2y - 6$

$x + 6 = 2y$

$\frac{1}{2}x + 3 = y = f^{-1}$

$x \rightarrow f(x)$	$f^{-1}(x)$
$0 \leftarrow -6$	0
$1 \leftarrow -4$	1
$2 \leftarrow -2$	2

Central to the idea of an inverse function: (f & g inverses)

$f(g(x)) = x$

ex $f = 2x - 6$

$f^{-1} = g = \frac{1}{2}x + 3$

$f(g(x))$

$2\left(\frac{1}{2}x + 3\right) - 6$

$x + 6 - 6$

x

ex $y = x^2 \quad y = \sqrt{x}$

$\sqrt{x^2} = x$

$f(g(x))=x$ $\quad g = f^{-1}$

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} x$$

$$f'(g(x)) \cdot g'(x) = 1$$

$$g'(x) = \frac{1}{f'(g(x))}$$

$g'(x) = \dfrac{1}{f'(g(x))}$

If $f(x) = x^5+2x-1$ then find $f^{-1}'(2)$.

① Set $f(x) = 2$.

② Find $f'(x)$.

③ Use formula

$x^5+2x-1 = 2$

guess & check $x=1$ ✓

$f: (1,2) \to f^{-1}(2,1)$

$f'(x) = 5x^4+2$

$f^{-1} \neq \dfrac{1}{f}$

$$f^{-1}'(2) = \frac{1}{f'(f^{-1}(2))}$$

$$f^{-1}'(2) = \frac{1}{f'(1)} = \frac{1}{7}$$

If f and g are inverses and $f(x)=4x^5+3x^3$, find $g'(7)$

didn't get to this one in class

① Set $f(x)=7$.

$4x^5+3x^3=7$

guess; check... $x=1$?

$4+3=7$ ✓ → $f:(1,7)$

$g:(7,1)$

② Find $f'(x)$

$f'(x)=20x^4+9x^2$

③ Use formula

$$g'(7)=\frac{1}{f'(g(7))} \rightarrow \frac{1}{f'(1)} \rightarrow \frac{1}{29}$$

The function f is defined by $f(x) = x^3 + 4x + 2$. If g is the inverse function of f and $g(2) = 0$, what is the value of $g'(2)$?

What to work on:

- work on related rates take home test
- prepare for retaking the diff eq or volume skills (hw, study, etc.)

- Non-testers: work on your roller coaster project due 5/4
- Testers: work on 2016 multiple choice set due 5/4

TESTERS:
We still good for the Saturday test??

ALL:
We still good for May 7th calculus party?