**Roller Coaster Project**

Due: 4/25

Goal: Design a roller coaster using a piecewise function with at least 5 functions that is everywhere continuous and differentiable. Turn in: functions, calculations showing continuity and differentiability at “handoff” points, 3D paper model, and poster-sized hand-drawn graph of roller coaster with each piece of the function clearly labeled.



Physics of Roller coasters: The starting and ending heights must be horizontal and equal. This horizontal part does not count as one of your 5 pieces. The “peaks” of a roller coaster must get progressively shorter. For ease of calculations, the points where the horizontal part meet the coaster do not need to be differentiable.

Example. Suppose our roller coaster starts up the “lift ramp” defined by $y=2x+50$ when $x\leq 0.$ The graph of this equation looks like this:



\*\*TI-TIP: Inequality signs can be found in 2nd Math (Test). Set domains such as $0\leq x\leq 6$ by typing in $(function)/(x\geq 0 and x\leq 6)$. “and” statements are also found in 2nd Math (Test) 🡪 Logic.

We need to develop a curve which is both continuous and differentiable at x = 0. Let’s suppose it is a downward parabola in the form of $f\left(x\right)=ax^{2}+bx+c.$

We know that in order to be continuous, then $f\left(0\right)=50,$ so $c=50,$ and $f\left(x\right)=ax^{2}+bx+50.$

In order for $f(x)$to be differentiable, then $f^{'}\left(x\right)=2ax+b$ must equal the slope of $2x+50$ at $x=0.$
So $2ax+b=2$ and when $x=0, b=2.$

So $f\left(x\right)=ax^{2}+2x+50.$ You can assign $a$ to be any value you wish. Since it is open downward, let $a=-1.$

So $f\left(x\right)=-1x^{2}+2x+50.$ Here is the graph and a zoom to show continuity and differentiability.

This is very steep, so adjust the *a* value to something more reasonable.

 

Another example.
Suppose you have Y2 as $-x^{2}+x+50, 0\leq x<6$ and you wish to connect it with another parabola. So $x=6$ is the point where your new parabola is going to begin.

Let us define our new parabola Y3 in the form $a(x-6)^{2}+b\left(x-6\right)+c.$ It will become apparent in a minute why. Our goal is to find $a, b,$ and $c.$

Now we want Y2 and Y3 to be continuous at $x=6.$
$Y2\left(6\right)=10$ by plugging in. $Y3\left(6\right)=c$. So $c=10.$

Now we want Y2 and Y3 to be differentiable at $x=6.$
$Y2^{'}=-2x+1$ so $Y2^{'}\left(6\right)=-11.$ $Y3^{'}=2a\left(x-6\right)+b$ so $Y3^{'}\left(6\right)=b.$

Since they must be the same, then $b=-11.$

Now $a$ can be anything you want. Since it will open up, $a$ must be positive. For now, try $a=1.$

So $Y3=1(x-6)^{2}-11\left(x-6\right)+10.$ Graph this for $x>6.$

Note that this goes way below the axis. So play around with the value of $a$, making it smaller, like 0.5 (You may have to play around with the window as well). You can adjust it to anything you want. Remember though that the $x$-axis really means nothing. Don’t think that the coaster has to start on the $x$-axis. To start it lower, just make your XMIN a smaller number. You might actually want to turn the axes off.

Hopefully you see why the parabola is in the $a(x-6)^{2}+b\left(x-6\right)+c$ format. It makes it far easier to work with because of the fact that when $x=6,$ many of the terms cancel out. So if your 4th curve was a parabola which needed to start at $x=10,$ your $Y4=a(x-10)^{2}+b\left(x-10\right)+c.$ If it was a cube, it would be in the form $a(x-10)^{3}+b(x-10)^{2}+c\left(x-10\right)+d.$

Finally, once you have your Y3, you may want to *foil* it out and collect terms so it looks “nicer”.

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In this project, you need at least 5 piecewise functions showing a possible roller coaster. You need to show the graph and calculations that show the function you generate is continuous and differentiable at the points where the pieces meet. You may use any type of function we have discussed including linear, trig, exponential, logarithms, inverse trig, and polynomial.

Products to turn in:

1. Paper with functions in piecewise form with full equations and calculations showing continuity and differentiability at transition points.
2. Poster sized graph of functions: posters should have each function equation included with the appropriate part of the roller coaster. Transition points should be clearly marked and given as an ordered pair. Be sure to name your coaster something exciting!
3. 3D Model made from graph.

**Remember:** (1) Roller coasters should start and end at same height. Use a horizontal line as loading area. This does not count as one of the curves. (2) Roller coasters do not go as high as their previous maximum. (3) Do you have what it takes to put in a loop?