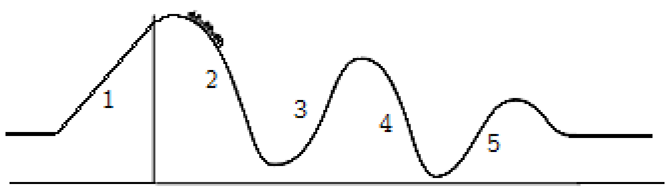
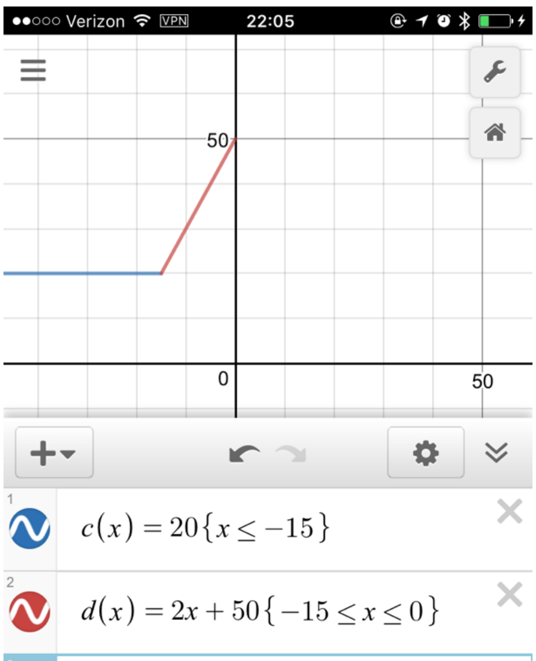
Roller Coaster Project DUE: 4/28

Goal: Design a roller coaster using a piecewise function with at least 5 functions that is everywhere continuous and differentiable. Turn in: functions, calculations showing continuity and differentiability at "handoff" points, 3D paper model, and poster-sized hand-drawn graph of roller coaster with each piece of the function clearly labeled.



Physics of Roller coasters: The starting and ending heights must be horizontal and equal. This horizontal part does not count as one of your 5 pieces. The "maxima" of a roller coaster must get progressively lower. For ease of calculations, the points where the horizontal part meet the coaster do not need to be differentiable.

Example. Suppose our coaster loads at $y = 20$ starts up the "lift ramp" defined by $y = 2x + 50$ when $x \leq 0$. These two lines intersect when $x = -15$, so the domain for the lift ramp is really $-15 \leq x \leq 0$

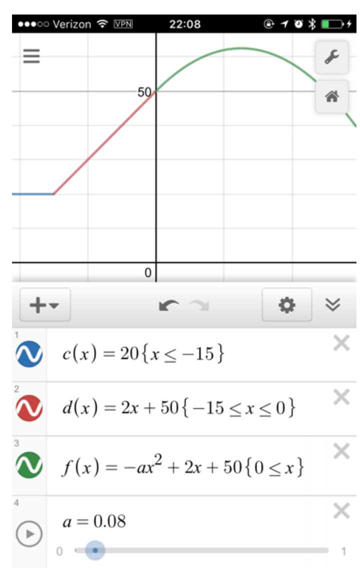


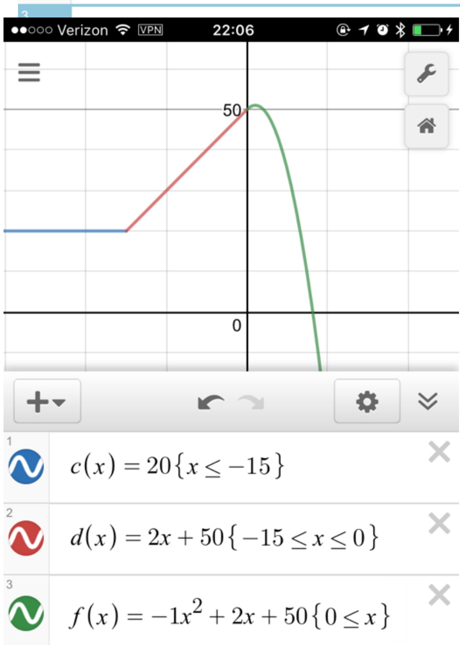
***Desmos Tips: Use braces for defining a function's domain. On a phone, pinch-to-zoom on the axes to adjust the x- and y-scales so that the functions are visible and reasonably sized.*

We need to develop a curve which is both continuous and differentiable at x = 0, which is where the curved part picks up from the lift ramp. Let's suppose it is a downward parabola in the form of $f(x) = ax^2 + bx + c$.

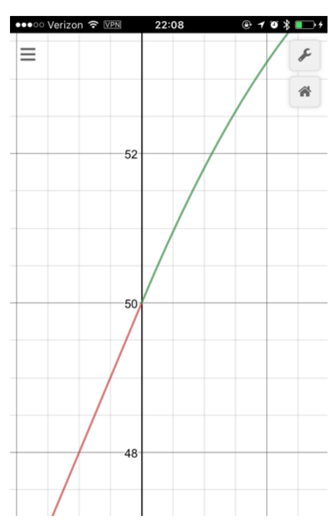
We know that in order to be continuous, then $f(0) = 50$, so $c = 50$, and $f(x) = ax^2 + bx + 50$.

In order for $f(x)$to be differentiable, then $f'(x) = 2ax + b$ must equal the slope of $2x + 50$ at $x = 0$.
So $2ax + b = 2$ and when $x = 0,\ b = 2$.





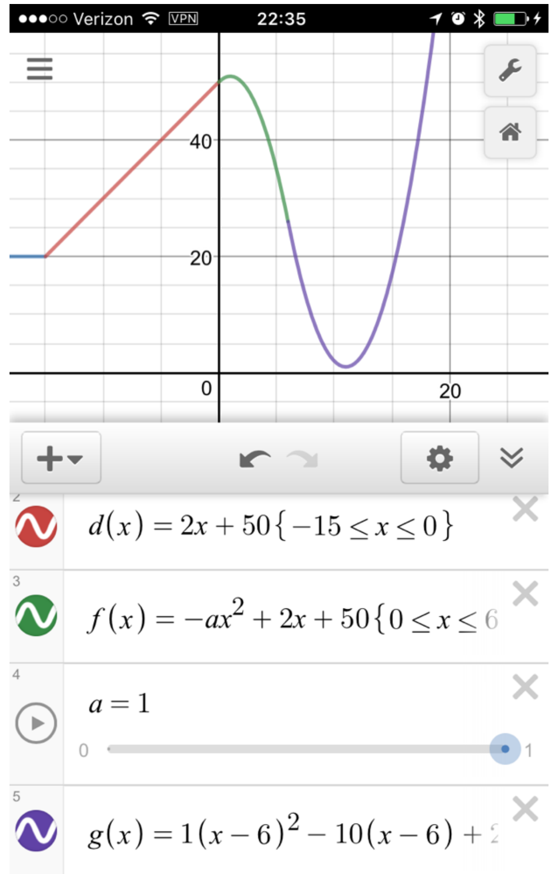
So $f(x) = ax^2 + 2x + 50$. You can assign a to be any value you wish. Since it is open downward, let $a = -1$.
So $f(x) = -1x^2 + 2x + 50$.



Here is the graph and a zoom-in to show continuity and differentiability.
This is very steep, so you can adjust the a-value to something more reasonable. Using sliders in Desmos is a good way to experiment.

Curve to curve handoff:

Suppose you have *f(x)* as $-x^2 + 2x + 50,\ 0 \leq\ x \leq 6$ and you wish to connect it with another parabola. So $x = 6$ is the point where your new parabola is going to begin. Note that 6 is an arbitrary point.



Let us define our new parabola *g(x)* in the form $a(x-6)^2 + b(x-6) + c$. It will become apparent in a minute why. Our goal is to find a, b, and c.

Now we want *f(x)* and *g(x)* to be continuous at $x = 6$.
$f(6) = 26$ by plugging in. $g(6) = c$. To be continuous, these should be equal, so $c = 26$.

Now we want *f(x)* and *g(x)* to be differentiable at $x = 6$.
$f'(x) = -2x + 2$ so $f'(6) = -10$. $g'(x) = 2a(x-6) + b$ so $g'(6) = b$.

Since differentiable functions have continuous derivatives, these should be equal, so $b = -10$.

Again a can be anything you want. Since it will open up, a must be positive. For now, try $a = 1$.
So $g(x) = 1(x-6)^2 - 11(x-6) + 10$. Graph this for $x \geq 6$.

Your functions may go beneath the x-axis, or maybe out of view. So play around with the value of a, making it smaller, like 0.5 (You may have to play around with the window as well).

Hopefully you see why the parabola is in the $a(x-6)^2 + b(x-6) + c$ format. It makes it far easier to work with because of the fact that when $x = 6$, many of the terms cancel out. So if your next curve was a parabola which needed to start at $x = 10$, your $h(x) = a(x-10)^2 + b(x-10) + c$. If it was a cubic function, it would be in the form $a(x-10)^3 + b(x-10)^2 + c(x-10) + d$.

Finally, once you have your *g(x)*, you should *foil* it out and collect terms so it looks "nicer". $g(x) = x^2 - 23x + 112$

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In this project, you need at least 5 curves showing a possible roller coaster. You need to give the graph and calculations that show the function you generate is continuous and differentiable at the points where the pieces meet. You may use any type of function we have discussed including trig, exponential, logarithms, inverse trig, and polynomial.

Products to turn in:

1. Document with function in piecewise form with full, simplified equations and complete calculations demonstrating continuity and differentiability at transition points.
2. Poster-sized graph of functions: posters should have each function included with the appropriate part of the roller coaster. Transition points should be clearly marked and given as an ordered pair. Color-coding curves and equations is suggested.
3. 3D paper model made from graph. Be sure to name your coaster something exciting!