(AP)

Exam Review

Function	Derivative
F(x)	Limit definition: F'(X) = $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
F(x) = c	0
F(x) = cx	c
F(x) = cx^n	cnx^{n-1}
F(x) = (G(x))^n	$n\cdot(G(x))^{n-1}\cdot G'(x)$ generalized power rule
H(x) = F(x) + G(x)	$F'(x) + G'(x)$
H(x) = F(x)*G(x)	$f'g + fg'$ Prod.
H(x) = F(x)/G(x)	$\frac{f'g - fg'}{g^2}$ Quotient rule
H(x) = F(G(x))	$F'(G(x))\cdot G'(x)$ Chain
F(x) = sin(x); cos(x); tan(x)	$\cos(x)$; $-\sin(x)$; $\sec^2(x)$
F(x) = csc(x); sec(x); cot(x)	$-\csc(x)\cot(x)$; $\sec(x)\tan(x)$; $-\csc^2(x)$
H(x) = e^(F(x))	$e^{f(x)}\cdot f'(x)$
H(x) = ln(F(x))	$\frac{1}{f(x)}\cdot f'(x)$

1.
Increasing: f'(x) $\underline{>}$ 0
Decreasing: f'(x) $\underline{<}$ 0
Concave up: f''(x) $\underline{>}$ 0
Concave down: f''(x) $\underline{<}$ 0

2.
Use the 4 terms above to describe each curve in two different ways:

NO CALC:

3. $f'=3$, $g' = dy/dx$
If $x^2 + \boxed{3xy} - y^2 = 2$, then find dy/dx.

$2x + 3y + 3x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3x - 2y) = -2x - 3y$

$\frac{dy}{dx} = \frac{-2x-3y}{3x-2y}$

4.
Find dy/dx if y = x^3e^x $f' = 3x^2$, $g' = e^x$

$3x^2e^x + x^3e^x$ → factor $\boxed{x^2e^x(3+x)}$

5. Find the *mx+b* form of the line tangent to the graph of *f(x) = 2x(1-3x)²* at the point (1, 8)

$f'(x) = 2(1-3x)^2 + 2x\cdot 2(1-3x)\cdot -3$

$f'(x) = 2(1-3x)^2 - 12x(1-3x)$

$f'(1) = 2(1-3)^2 - 12(1)(1-3)$

$2(-2)^2 - 12(-2)$

$2\cdot 4$

$f'(1) = 8 - -24 = \underline{32}$

$y - y_1 = m(x - x_1)$

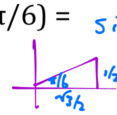
$y - 8 = 32(x-1)$

$y - 8 = 32x - 32$

$\boxed{y = 32x - 24}$

6. If f(x) = cos(x), then f'(π/6) =

sine → y-coordinate



$f'(x) = -\sin(x)$

$f'(\pi/6) = -\boxed{\sin(\pi/6)} = -1/2$

7. f'(3) for y = $\sqrt{3x}$ is…

$y = (3x)^{1/2}$

$y' = \frac{1}{2}(3x)^{-1/2}\cdot 3$

$y' = \frac{1}{2\sqrt{3x}}\cdot 3 = \frac{3}{2\sqrt{3x}}$

@ $x = 3$ → $\frac{3}{2\sqrt{3\cdot 3}} = \frac{3}{2\cdot 3} = \frac{3}{6} = 1/2$

Velocity $= 0 \Rightarrow v(t)=0$

8. A particle's position is given by $x(t) = 2t^3-6t^2-18t-2$. For $t \geq 0$, when is the particle at rest?

$x'(t) = v(t) = 6t^2 - 12t - 18 = 0$

$6(t^2-2t-3)=0$

$6(t-3)(t+1)=0$

$t=3,\ t=-1$

$t=3$

9. If $y = 2\sin(x/2)$, then $\frac{d^2y}{dx^2} =$

$\frac{dy}{dx} = 2\cos\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$\frac{d}{dx}\frac{dy}{dx}\ \frac{d}{dx}\left(\cos\left(\frac{x}{2}\right)\right)$

$\frac{d^2y}{dx^2} = -\sin\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$-\frac{1}{2}\sin\left(\frac{x}{2}\right)$

See if f' is continuous.

Continuous? left = right

$2^3 = 2(2)+4$

$8 = 8$

Cont.

Diff?

$f'(x) = \begin{cases} 3x^2, & x<2 \\ 2, & x>2 \end{cases}$

left = Right

$3\cdot 2^2 = 2$

$12 = 2$

No.

10. At x=2, the function given by $f(x) = \begin{cases} x^3, & x \leq 2 \\ 2x+4, & x>2 \end{cases}$ is:

undefined / continuous but not diff / diff but not continuous / neither continuous nor diff / both cont and diff

(Impossible)

11.
$$\lim_{h\to 0}\frac{\sin 3(x+h) - \sin(3x)}{h} =$$

$f(x) = \sin(3x)$

$f'(x) = \boxed{3\cos(3x)}$

15

$\frac{dr}{dt} = 0.2$ (positive, b/c "increasing")

$\boxed{\frac{dA}{dt} = ?}$ @ $C = 20\pi$

Well, $C = 2\pi r = 20\pi$

So $r = 10$

$\frac{d}{dt}A = (\pi r^2)\frac{d}{dt}$

$\frac{dA}{dt} = \pi\cdot 2r\,\frac{dr}{dt}$

$\frac{dA}{dt} = 2\cdot\pi\cdot 10\cdot\frac{2}{10}$

4π

12. If $\lim_{x\to 2} f(x) = 8$, then which of the following must be true:

Roads meet.

I. f is continuous at x = 2 → No, bridge could be elsewhere.

II. f is differentiable at x = 2 → Not necessarily continuous, so diff is unknown.

III. f(2) = 8 → Bridge could be elsewhere.

None / II only / III only / I and III only / I, II, and III

13. Write a rational function that has y=0 as an asymptote.

any function with polynomials, bottom degree bigger.

ex. $f(x) = \frac{x+5}{x^2+1}$

14. $\lim_{x\to 0}\frac{\sin(2x)}{2x} =$

As $x\to 0$, $2x \to 0$, so $\lim_{2x\to 0}\frac{\sin(2x)}{2x} = 1$

15. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

a. 0.04π m²/sec b. 0.4π m²/sec c. 4π m²/sec d. 20π m²/sec e. 100π m²/sec

Yes Calculator

16. If f is continuous, and $f(x) = \frac{x^2-25}{x-5}$ for all $x\neq 5$, then $f(5) =$

0/1/2/5/10

$f(x) = \frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}} = x+5$

$f(x) = x+5$

$f(5) = 5+5 = 10$

17. Let f and g be two differentiable functions. Describe how you would find the x-value where they have parallel tangent lines. (Calculator active) find $f'(x)$ and $g'(x)$.

put them in Calculator, Y_1 and Y_2. Find pt of intersection. $\boxed{2nd}$ + $\boxed{Calc}$

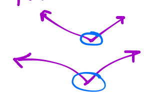
18. You are given a derivative function. How do you find the number of critical values on the interval (0,5)?

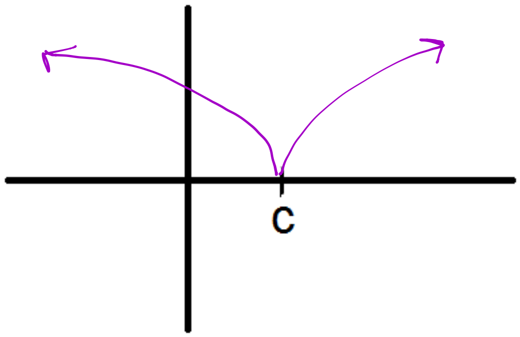
put $f'(x)$ into Y_1 in calculator. Adjust x-values of window to ≈ (0,5). I suggest [-1, 6].

Critical numbers: $f'=0$ or undefined. Count x-intercepts

19. Sketch a graph below where f is continuous at x=c, (c,f(c)) is an absolute minimum, but f is $\boxed{\text{not differentiable}}$ at x=c.

Not smooth.





20. Find the equation of a line tangent to f(x) = $2x^4-3x^2+4x$ at the point where f'(x) = 1.

$f'(x) = 8x^3-6x+4=1$

$-1 \quad -1$

$8x^3-6x+3=0$

use calculator: Graph find zeros.

$x=-1.05$

$y=f(-1.05)=2(-1.05)^4-3(-1.05)^2+4(1.05) \approx 3.323$

this is m. need (x,y).

$(1.05, 3.323)$

$y-3.323 = 1(x-1.05)$

$y = x-1.05+3.23$

$\boxed{y=x+2.273}$

21. f'(x) = $2e^x - 12x$. Where does f have a relative minimum?

Graph f' in Y_1 in calculator.

find where f' crosses x-axis from neg → pos

like this Y_1

. Use $\boxed{2nd}$ + $\boxed{Calc}$ + ZERO.

22. Let f(x) = $\sqrt{x}$. If the rate of change of f at x =c is four times the rate of change at x = 1, then c=

$f=x^{1/2}$

$f'=\frac{1}{2\sqrt{x}}$

$f'(c)=\frac{1}{2\sqrt{c}} = 4 \cdot f'(1)$

$\frac{1}{2\sqrt{1}} = \frac{1}{2}$

$\frac{1}{2\sqrt{c}} = 4 \cdot \frac{1}{2}$

$\frac{1}{2\sqrt{c}} = 2$

$\frac{1}{2\sqrt{c}} \times \frac{2}{1}$

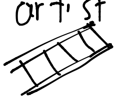
$4\sqrt{c}=1$

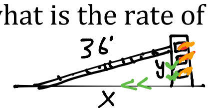
$(\sqrt{c}=1/4)^2$

$c=1/16$

23. The top of a 36 foot ladder is sliding down a vertical wall at a constant rate of 5 ft per minute. When the top of the ladder is 8 feet from the ground, what is the rate of change in feet per minute of the distance between the bottom of the ladder and the wall?

I'm an artist →





$\frac{dy}{dt} = -5$ (negative b/c down)

$\frac{dx}{dt} = ?$ @ $y=8$

$x^2+y^2=36^2$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

need

$y=8$

$\Rightarrow x^2+8^2=36^2$

$x^2=1232 \rightarrow x \approx 35.1$

$2(35.1)\frac{dx}{dt}+2(8)(-5)=0$

$\frac{(70.2)\frac{dx}{dt}}{70.2} = \frac{80}{70.2}$

$\frac{dx}{dt} \approx 1.140$

24. Find the value of *c* that satisfies the conclusion of the Mean Value Theorem for the function f(x) = x^2-2x+3 on the interval [1, 3].

MVT: Avg = Instant

$f'(x)=2x-2$

Avg: $\frac{f(3)-f(1)}{3-1} = \frac{6-2}{2} = \frac{4}{2} = 2 = 2x-2$

$+2 \quad +2$

$4=2x$

$\boxed{x=2}$

$c=2, \in [1,3]$

25. The function f(x) = $\begin{cases} x^2-2x+1, & x \le 2 \\ ax+b, & x > 2 \end{cases}$ is differentiable everywhere. Find *a* and *b*.

implies Continuity

Continuous

left = Right

$2^2-2\cdot2+1=2a+b$

$1=2a+b$

$f'(x)=\begin{cases} 2x-2, & x<2 \\ a, & x>2 \end{cases}$

assume f' cont: left = right

$2(2)-2=a$

$4-2=a$

$2=a$

$1=2(2)+b$

$1=4+b$

$b=-3$

$\boxed{a=2,\ b=-3}$