Critical Numbers

→ Take derivative; set $= 0$, solve.

Or, when derivative is undefined.

$$f'(x) = \frac{}{} = 0$$

ex/ $\frac{0}{7} = 0$

$\frac{0}{8} = 0$

11)

$$f(x) = \frac{-9x}{x^2+9} \quad \begin{matrix} f \\ g \end{matrix}$$

$$f' = -9 \qquad g' = 2x$$

$$f' = \frac{-9\cdot(x^2+9) - (-9x)(2x)}{(x^2+9)^2}$$

$$\frac{-9x^2 - 81 - (-18x^2)}{(x^2+9)^2} \rightarrow \frac{9x^2 - 81}{(x^2+9)^2}$$

$$9x^2 - 81 = 0$$

$$\sqrt{(x^2+9)^2} = \sqrt{0}$$

$$9(x^2-9) = 0 \qquad 9x^2 = 81$$

$$9(x+3)(x-3) = 0 \qquad \sqrt{x^2} = \sqrt{9}$$

$$x^2 + 9 = 0$$

$$\sqrt{x^2} = \sqrt{-9}$$

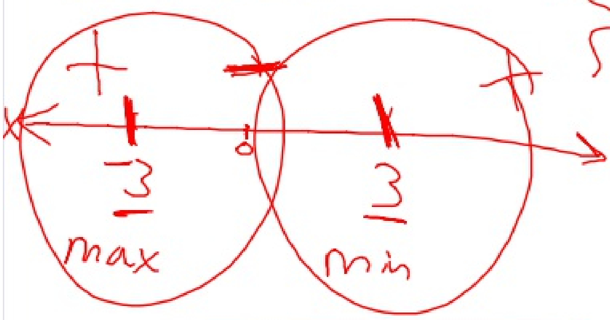
???

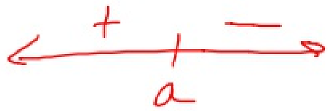
$x=3$, $x=-3$

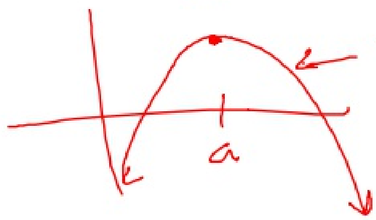
critical numbers

$$x = 3, -3$$

$$\pm 3i$$







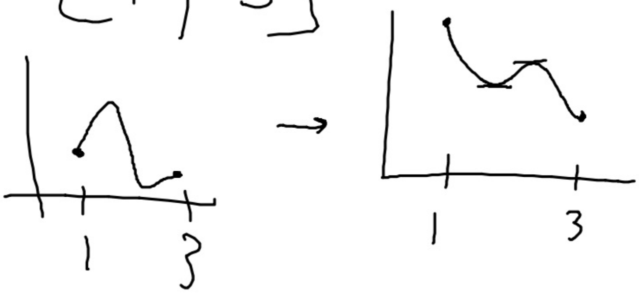
$$-ax^2 + bx + c$$

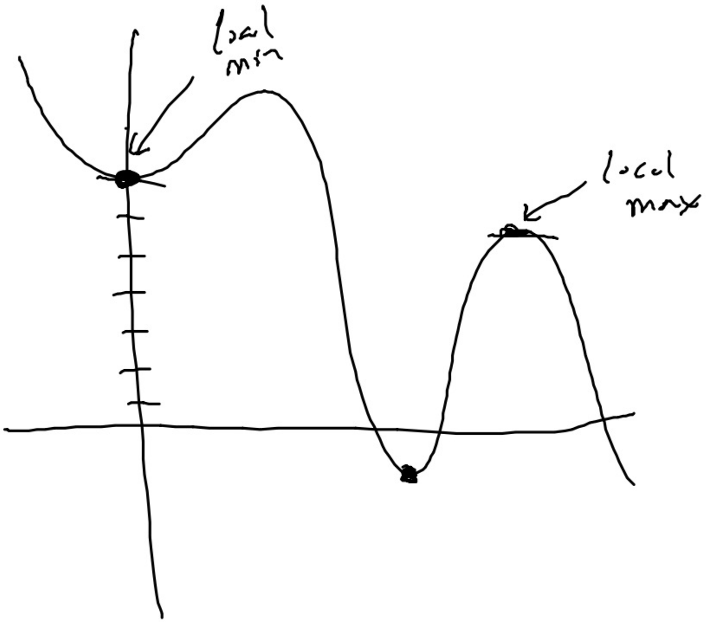
$$-\frac{1}{2}s^2 + 3s - 3/2$$

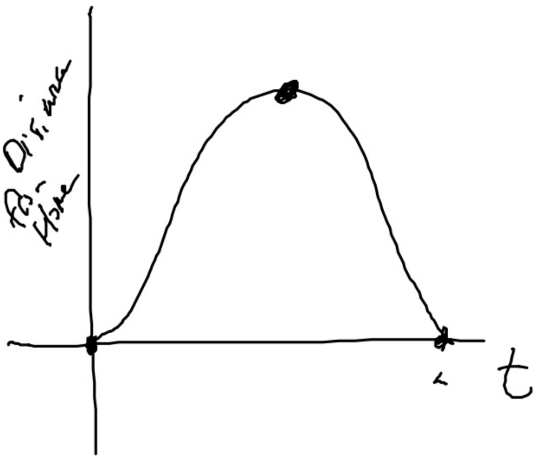
Absolute Extrema

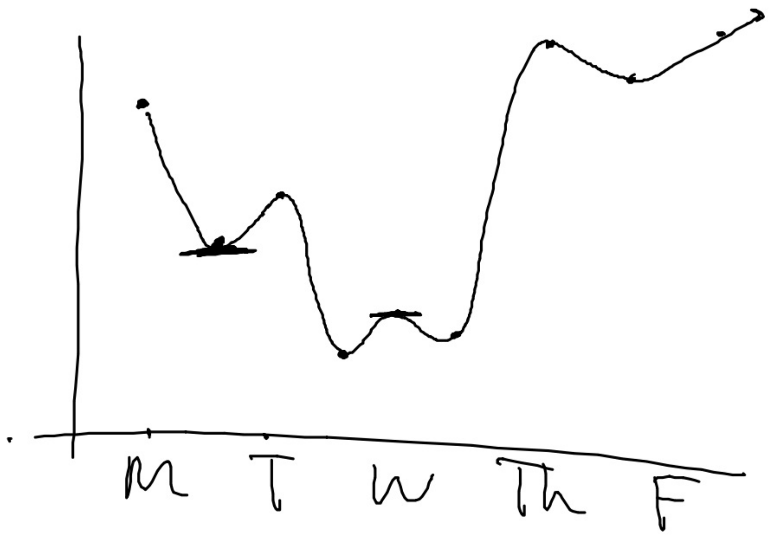
- Absolute extremes occur either
  at ① local maxes/mins
  or ② at endpoints of an interval.

[1, 3]




local min
local max


t



[M, F]: abs min: Tues night

abs max: Fri Night.

ex/ Find the absolute extrema of

$$f(x) = 2x^3 - 6x^2 + 6x - 2$$

over $[1, 4]$.

$$f'(x) = 6x^2 - 12x + 6 = 0$$

$$6(x^2 - 2x + 1) = 0$$

$$6(x-1)(x-1) = 0$$

$x = 1$ critical number/value

local max/min occur @ critical values where f' changes sign.

x		1	
f'	$+$		$+$

→ (no local extrema)

→ $f(4) = 54$

$f(1) = 0$

f has abs. max @ $x = 4$; f has abs min @ $x = 1$.

ex/ $f(x) = 3x^2 - 2x + 1$ over $[-1, 3]$.

$f' = 6x - 2 = 0$

$6x = 2$

$\underline{x = \frac{1}{3}}$ Critical value.

f' — +

x 0 $\frac{1}{3}$

★★★ Find local min. $\boxed{@\ x = \frac{1}{3}}$

4 values @ $x = -1$ $f(-1) = 6$

$x = 3$ $f(3) = 22$

$x = \frac{1}{3}$ $f(\frac{1}{3}) = \frac{2}{3}$

f has abs min @ $x = \frac{1}{3}$

abs max: @ $x = 3$.